Pearson Edexcel

Pearson Edexcel
Level 3 Advanced GCE in Mathematics (9MA0)

May/June 2023 Exemplar
9MA0-01 A level Mathematics
Paper 01: Pure Mathematics

Senior Examiner's feedback on student responses

Contents

Pearson: helping people progress, everywhere
Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

About this booklet

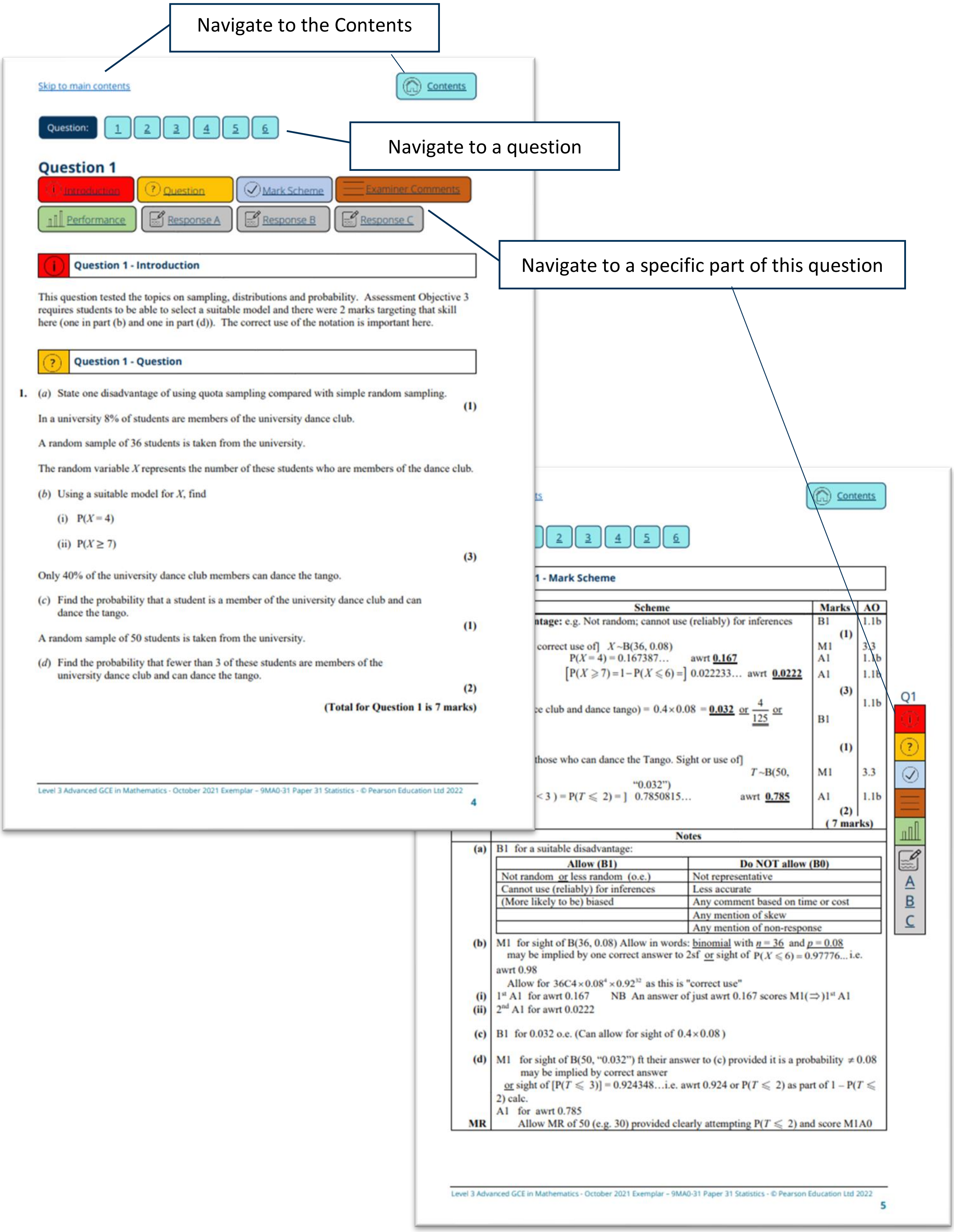
This document has been produced to support mathematics teachers delivering the GCE Mathematics specification.

This document looks at questions from the 9MA0-01 A level Mathematics Paper 1: Pure Mathematics June 2023 examination paper. It shows real student responses to these questions, and how the examining team follow the mark schemes to demonstrate how the students are awarded the marks. For the mark scheme notes and details of alternative methods please see the full mark scheme for this question paper on our website.

For 2023, the approach all exam boards have taken to grading was to return to pre-pandemic grading, while giving students protection against any impact of disruption. Results in summer 2023 therefore will be far more in line with summer exams that were sat in 2019, but lower than in 2022, when grades awarded were based on a mid-point between 2019 and 2021 outcomes. For more information please read our 'Understanding grade boundaries 2023' document.

* The question level performance data is there to give an indication only of how students performed, on each question, in the context of sitting the entire exam paper and is not an indication of how students may perform sitting a question in isolation. The performance data of this series doesn't represent a normal series due to small number of entries.

How to use this booklet



General Examiner Feedback

This was the second set of papers taken by the full cohort of candidates since improvements were made to the accessibility of questions. The aim was to improve the exam experience of candidates, and feedback regarding their experience was largely positive. There were 15 questions on the paper, which was one less than last year, however the greatest number of marks on any part of any question was 6.

With the improvements designed to focus on helping candidates to get off to a good start, it was pleasing that questions 2, 3 and 5 were appropriately placed in the first five. In fact, typically candidates scored full marks on Questions 1, 2, 4, 5, 6, 8, 9, 11 and 12 and usually candidates scored 2 out of 3 on Question 3. Question 12 was one of the most successfully answered questions on the paper and Question 8 on radians was a high scoring question for many. Question 1, however, seemed to prove more challenging that it was intended to be, often due to the approach taken by some candidates.

The questions had plenty of parts which were not reliant on each other, or later parts using an established result from a "show that" part of a question e.g. Question 8 part (c) could be answered with the aid of the established results in part (a) and part (b). Question 15 was the final question on the paper, and this was the most challenging (and longest) which provided suitable challenge to the most able candidates. This question was demanding, but it was pleasing to see many candidates able to make progress on part (a) and many of all abilities attempted parts (c) and (d) to a pleasing level of success. It was rare for candidates not to score at least one mark on Question 15. Time did not appear to be an issue for candidates to demonstrate what they could do over the whole paper.

The presentation of solutions was generally good, although a number of candidates still do not show all the steps in their working or indicate what they are trying to do. This is particularly an issue when their answer is incorrect but there is no method visible to determine whether the candidate had made a slip in their method or not. Alternatively, it can be an issue when there are many attempts, with a lot of incorrect approaches and no indication as to what should be marked.

Another increasing trend is the use of calculators to solve quadratic equations (e.g. Question 2) and simultaneous equations (e.g. Question 5). This is an acceptable approach and is encouraged. Candidates should be reminded, however, that they need to show sufficient steps in their method, if they are to gain full credit for their solutions as, for example, solving a cubic or quartic would likely require some factorising first to achieve a quadratic, which then the use of a calculator would be appropriate for the quadratic. The advice at the top of some questions indicates where it is even more important to show the full method to the solution, and the extent to which calculators can be used. It was noticed on several occasions that some candidates write down an incorrect quadratic formula, demonstrating a lack of understanding of a key process, but are then able to produce the correct solution via use of a calculator.

Question 1

Question 1 - Introduction

This was a short 4-mark question on integration. Candidates were expected to multiply out the brackets to achieve a sum of terms which they could then integrate separately. The constant of integration was also required as part of their final answer. There were other methods of integration which could be used, although these were less efficient for what was required.

Question 1 - Question

1. Find

$$\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\,\mathrm{d}x$$

writing each term in simplest form.

(4)

Total for Question 1 is 4 marks)

Question 1 - Mark Scheme

Question	Scheme	Marks	AOs
1	$x^{\frac{1}{2}}(2x-5) = \ldots x^{\frac{3}{2}} + \ldots x^{\frac{1}{2}}$ or $\dfrac{x^{\frac{1}{2}}(2x-5)}{3} = \dfrac{\ldots x^{\frac{3}{2}} + \ldots x^{\frac{1}{2}}}{3}$	M1	1.1b
	$\dfrac{2x^{\frac{3}{2}}}{3} - \dfrac{5x^{\frac{1}{2}}}{3}$	A1	1.1b
	$\displaystyle\int \dfrac{2x^{\frac{3}{2}}}{3} - \dfrac{5x^{\frac{1}{2}}}{3}\,\mathrm{d}x = \ldots x^{\frac{5}{2}} \pm \ldots x^{\frac{3}{2}} \ (+c)$	dM1	1.1b
	$\dfrac{4}{15}x^{\frac{5}{2}} - \dfrac{10}{9}x^{\frac{3}{2}} + c$	A1	1.1b
		(4)	
			(4 marks)

Question 1 - Examiner Comments

This was an accessible question for virtually all candidates. The fractional index had already been given to avoid any misconceptions. However, for the first question on the exam paper, this was frequently done poorly compared to previous years. Candidates may have anticipated a tougher start to the paper and wanted to demonstrate one of the more advanced integration techniques.

Candidates who spotted that you needed to expand the brackets usually went on to score full marks. The most common reason for losing the final answer mark was forgetting the constant of integration.

Many candidates failed to notice the simpler method in the main mark scheme and instead attempted to use integration by parts, with either part taken as u. If a candidate chose to set $u = 2x - 5$, they did not always expand the brackets in their integration formula to achieve a form which was ready to integrate where the indices had been combined correctly. Candidates who set $u = x^{\frac{1}{2}}$ were more successful, as there was less working to do to allow access to the marks. Candidates who chose to do integration by parts did not always combine the terms in their answer, losing the final mark. Many candidates would divide both their u and v' by 3, leading to final answers that were $\frac{1}{3}$ as large as they should have been.

Integration by substitution was occasionally seen, but rarely successfully completed. Most attempts involved setting $u = 2x - 5$, and this resulted in a tougher integral that required further work; candidates rarely progressed to a point which would have been worthy of a mark.

Q1

A

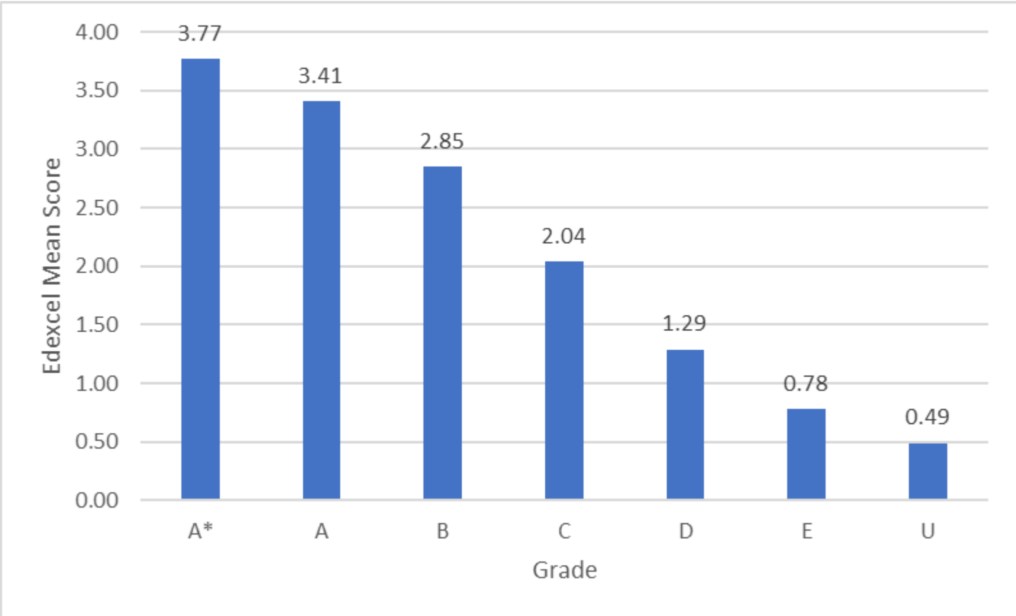
B

C

Across the various methods, a common error seen was initially to factorise out the $\frac{1}{3}$, work through the solution without it, but then to forget to reintroduce it. This error meant that neither of the two accuracy marks were available.

Question 1 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
2.57	4	64%	3.77	3.41	2.85	2.04	1.29	0.78	0.49



Question 1 - Response A

11) $\int \frac{x^{1/2}(2x-5)}{3}\,dx$

$x^{1/2}\;(2x-5)$

$\frac{2x^{1/2}-5x^{1/2}}{3}$

$\int \frac{2x^{1/2}}{3} - \frac{5x^{1/2}}{3}$

$2x^{1/2}\ln|3| - 5x^{1/2}\ln|3|$

$-3x^{1/2}\ln(1)$

1/4 marks

M1: Attempts to multiply out the brackets of the numerator and writes the expression as a sum of terms with indices. Scored for one correct index.

A0: Incorrect sum of terms. The power is incorrect on the first term of the numerator.

dM0: Does not increase the power by one on an x^n term where n is a fraction.

A0: Follows A0

Examiner comments

Candidates quite often did not set out their solution in a particularly clear and methodical way and this candidate also confused integrating $\frac{1}{x}$ with integrating $\frac{1}{3}$ in a similar way.

Question 1 - Response B

(4)

$$u = \frac{1}{3}x^{1/2} \qquad v' = \frac{1}{3}(2x-5)$$

$$= \frac{2}{3}x - \frac{5}{3}$$

$$u' = 1/6x^{-\frac{1}{2}}$$

$$v = \frac{1}{3}x^2 - \frac{5}{3}x$$

$$\left(\frac{1}{3}x^2 - \frac{5}{3}x\right)\left(\frac{1}{3}x^{1/2}\right) - \int\left(\frac{1}{6}x^{-1/2}\right)\left(\frac{1}{3}x^2 - \frac{5}{3}x\right)$$

$$\frac{1}{3}x^{5/2} - \frac{5}{9}x^{3/2} - \int \frac{1}{18}x^{3/2} - \frac{5}{18}x^{1/2}$$

$$\frac{1}{3}x^{5/2} - \frac{5}{9}x^{3/2} - \left[\frac{1}{45}x^{5/2} - \frac{5}{27}x^{3/2}\right]$$

$$\frac{1}{3}x^{5/2} - \frac{5}{9}x^{3/2} - \frac{1}{45}x^{5/2} + \frac{5}{27}x^{3/2}$$

$$\frac{14}{45}x^{5/2} - \frac{10}{27}x^{3/2} + c$$

2/4 marks

M1: Applies integration by parts correctly to achieve the form

$$\int x^{\frac{1}{2}}(2x-5)\,dx = ...x^{\frac{1}{2}}(x^2-5x) - \int ...x^{\frac{3}{2}} \pm ...x^{\frac{1}{2}}\,dx$$

Candidates were expected to multiply out the brackets of the terms which still required integrating to score this mark, although errors dealing with coefficients were condoned.

A0: Does not achieve the correct intermediate stage.

dM1: Increases the power by one on an x^n term where n is a fraction.

A0: Follows A0

Examiner comments

Using integration by parts was attempted by a number of candidates, but usually there were errors either in applying the formula or dealing with the coefficients. Some candidates also did not then simplify by collecting terms.

Question 1 - Response C

$$= \frac{2x^{3/2} - 5x^{1/2}}{3}$$

$$2x^{3/2} = \frac{2x^{5/2}}{5/2} = 4/5\, x^{5/2}$$

$$5x^{1/2} = \frac{5x^{3/2}}{3/2} = 10/3\, x^{3/2}$$

$$= \frac{4/5\, x^{5/2} - 10/3\, x^{3/2}}{3}$$

3/4 marks

M1: Attempts to multiply out the brackets of the numerator and writes the expression as a sum of terms with indices. This is scored on their first line of working.

A1: $\dfrac{2x^{\frac{3}{2}} - 5x^{\frac{1}{2}}}{3}$ (also scored on their first line of working)

dM1: Increases the power by one on an x^n term where n is a fraction. This is scored on their second line of working. It was acceptable to integrate terms on separate lines.

A0: Incorrect

Examiner comments

A number of candidates either forgot the constant of integration or did not write each term in simplest form. Fractions within fractions were not accepted as simplified coefficients.

Q1

A

B

C

Question 2

Question 2 - Introduction

This was a straightforward question requiring candidates to typically use the factor theorem in part (a). If they were unable to do this, they were still able to find the value of *a* in part (b)(i) using the given equation and, as they were told in the question that *a* is a positive constant, candidates could proceed to solve a cubic equation in (b)(ii). The question clearly stated that all stages of working must be shown and that solutions relying entirely on calculator technology were not acceptable.

Question 2 - Question

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 4x^3 + 5x^2 - 10x + 4a \qquad x \in \mathbb{R}$$

where a is a positive constant.

Given $(x - a)$ is a factor of $f(x)$,

(*a*) show that

$$a(4a^2 + 5a - 6) = 0$$

(2)

(*b*) Hence

(i) find the value of a

(ii) use algebra to find the exact solutions of the equation

$$f(x) = 3$$

(4)

(Total for Question 2 is 6 marks)

Question 2 - Mark Scheme

Question	Scheme	Marks	AOs
2(a)	$(\mathrm{f}(a)=)\ 4a^3+5a^2-10a+4a=0 \Rightarrow a(....)=0$	M1	3.1a
	$a(4a^2+5a-6)=0$ *	A1*	1.1b
		(2)	
(b)(i)	$a=\frac{3}{4}$	B1	2.2a
(ii)	$4x^3+5x^2-10x+4\times"\frac{3}{4}"=3 \Rightarrow 4x^3+5x^2-10x\ (=0)$	M1	1.1b
	$x=0$	B1	1.1b
	$x=\frac{-5\pm\sqrt{185}}{8}$	A1	1.1b
		(4)	
		(6 marks)	

Question 2 - Examiner Comments

This question was accessible and generally well answered, with a large number of candidates gaining full marks with concise solutions. Some, however, did not pay heed to the instruction to show all stages of their working.

The vast majority of candidates correctly identified that part (a) related to an application of the factor theorem, substituted in a and were able to reach the required result. However, many lost marks for not including " $=0$" until the final line. In a small number of cases, " $=0$" was omitted completely and, in these cases, candidates were awarded no marks. Amongst the unsuccessful candidates, the majority had attempted algebraic long division using $(x-a)$ or carried out an attempt at factorisation via inspection, but had often abandoned their attempts, presumably unsure of how this would lead to the required result.

Very few successfully attempted long division and set the remainder equal to 0. Factorisation using inspection then solving two simultaneous equations by equating like terms was rarely seen as a method.

In part (b)(i), most candidates had no problems solving the cubic, getting the values 0, $\frac{3}{4}$ and -2 for a. However, many did not take note of the fact that a was positive, and that the question asked for the **value** of a, so lost the mark for not rejecting $a = 0$ and $a = -2$. There was confusion as to the reasons for rejecting values. Some thought that a had to be an integer so rejected $\frac{3}{4}$. Some chose -2 as their solution giving spurious reasons such as “a is a real number.”

Of those who did not identify $a = \frac{3}{4}$ as the only valid answer, many recovered by opting to use only $a = \frac{3}{4}$ in part (b)(ii).

Part (b)(ii) was well done by those who showed every step of their working, but some failed to reject the values of a that were not needed.

Once the cubic was obtained by substituting $a = \frac{3}{4}$ and setting $f(x) = 3$, those who solved the cubic without factorising to $x(4x^2 + 5x - 10)$ lost the final A mark. Candidates were asked to show all stages of their working and not proceeding as far as the quadratic factor was penalised. Many gave the exact solutions followed by rounded decimal solutions, or even just the decimal solutions, so were not paying heed to the question asking for exact solutions.

There were some who were unable to solve the cubic or the quadratic, having taken out a factor x outside the brackets. These were mainly cases where they tried to complete the square or their value for a meant that they could make no further progress. Some did not collect terms on one side with 0 on the other side, and having taken out a factor of x, solved their quadratic ignoring the constant on the other side.

The solution $x = 0$ was omitted by many candidates due to division by x instead of using factorisation. Others rejected this particular solution, if it was found, possibly confusing the restriction that a was positive rather than x. Those who did reach the correct quadratic factor were usually successful in finding the two exact roots for the A1 mark.

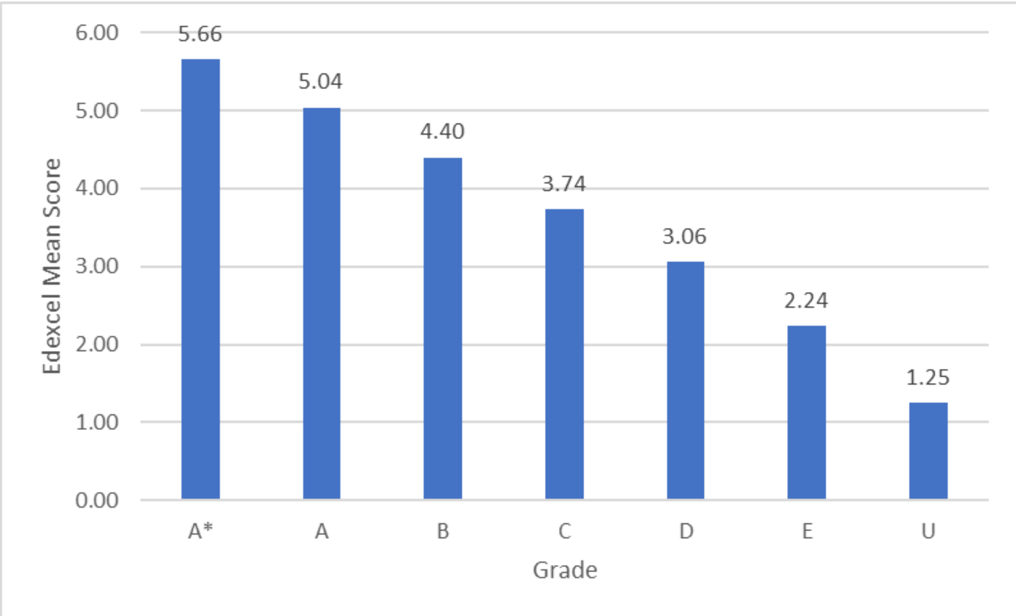
A common incorrect approach was to find $f(3)$ rather than setting $f(x) = 3$. This approach gained no credit.

? Skip to Main Contents

Question: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Question 2 - Performance

Mean score	Max score	Mean %	Edexcel averages: mean scored by candidates achieving grade: A*	A	B	C	D	E	U
4.20	6	70%	5.66	5.04	4.40	3.74	3.06	2.24	1.25



Q2

A
B
C

Question 2 - Response A

a) $f(a) =$
$= 4(a)^3 + 5(a)^2 - 10(a) + 4a$
$= 4a^3 + 5a^2 \cancel{-10a} - 6a$
$= a(4a^2 + 5a - 6) = 0$

b)i) $a = 0$ $\quad 4a^2 + 5a - 6 = 0$

$a = \frac{3}{4}$ $\quad \boxed{a = -2 \text{ ONLY}}$

ii) $4x^3 + 5x^2 - 10x + 4a = 3$
$4x^3 + 5x^2 - 10x - 8 = 3$
$4x^3 + 5x^2 - 10x - 11$

1/6 marks

Part (a)

M1: Attempts $f(a) = 0$ leading to an equation in a only and attempts to take a factor out.
(scored on the final line of (a) where the $= 0$ appearing is condoned for the method mark)

A0*: Achieves the given answer but the $= 0$ does not appear until they write the given answer.

Q2

i ?

A

B

C

Part (b)(i): Mark (i) and (ii) together

B0: Deduces that $a = -2$ which is incorrect.

Part (b)(ii)

M0: Does not substitute a positive value for *a* into f(*x*)

B0: $x = 0$ is not seen.

A0: Follows M0

Examiner comments

Candidates who selected the negative root for (b)(i) were unlikely to score any marks in (ii) as typically they would not have achieved a cubic where $x = 0$ would be a solution.

Q2

i

?

A

B

C

Question 2 - Response B

(4)

if $(x-a)$ is a factor then $f(a)$ is a factor

$f(a) = 4(a)^3 + 5(a)^2 - 10(2a) + 4a$

$= 4a^3 + 5a^2 - 10a + 4a$

$4a^3 + 5a^2 - 6a$

$a(4a^2 + 5a - 6)$

$a((4a-3)(a+2))$

b) $a = 0$ or $a = \frac{3}{4}$ or $\cancel{a = -2}$

$x \neq -2$

a is a positive constant.

$f(x) = 4x^3 + 5x^2 - 10x + 3$

$4x^3 + 5x^2 - 10x + \cancel{3} = 3$

$4x^3 + 5x^2 - 10x = 0$

$x(4x^2 + 5x - 10)$

$a = 4 \quad b = 5 \quad c = -10$

$x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 4 \cdot -10}}{2 \cdot 4}$

$x = \frac{-5 + \sqrt{185}}{8}$ or $x = \frac{-5 - \sqrt{185}}{8}$

3/6 marks

Part (a)

M0: Attempts f(a) but never sets equal to 0

A0*: Follows M0

Part (b)(i): Mark (i) and (ii) together

B1: Deduces that $a = \frac{3}{4}$ only. They circled their answer, but it can also be implied by their resultant cubic.

Part (ii)

M1: Attempts to substitute $a = \frac{3}{4}$ into f(x), sets their $\text{f}(x) = 3$ and collects terms on one side.

B0: $x = 0$ not found.

A1: $\frac{-5 \pm \sqrt{185}}{8}$ or exact equivalent after sufficient working seen. In this case they took a factor of x out leading to the correct quadratic seen so this final mark can be scored for correct solutions.

Examiner comments

For this approach in part (a) it was important to demonstrate knowledge of the factor theorem by setting $\text{f}(a) = 0$ before achieving the given result.

Question 2 - Response C

a) $x=a$

$f(a) = 4(a^3)+5(a^2)-10(a)+4a = 0$

$4a^3+5a^2-6a=0$

$a(4a^2+5a-6)=0$

b) i) $a=0, \frac{3}{4}, -2$

$a=\frac{3}{4}$

ii) $3=4x^3+5x^2-10x+4(\frac{3}{4})$

$0=4x^3+5x^2-10x$

$x=\frac{-5+\sqrt{185}}{8}, 0, \frac{-5-\sqrt{185}}{8}$

5/6 marks

Part (a)

M1: Attempts $f(a)=0$ leading to an equation in a only and attempts to take a factor of a out. (scored on their final line of (a))

A1*: Achieves the given answer with no errors including brackets. The setting equal to zero is seen before the given answer is achieved.

Part (b)(i): Mark (i) and (ii) together

B1: Deduces that $a=\frac{3}{4}$ only. Also implied by their resultant cubic.

Part (ii)

M1: Attempts to substitute $a=\frac{3}{4}$ into f(x), sets $f(x)=3$ and collects terms on one side

scored on line 2 of (ii)

B1: $x=0$

A0: Correct answer but quadratic factor not seen. They cannot proceed directly from the cubic to the roots.

Examiner comments

The question stated at the top that candidates must show all stages of their working and that solutions relying entirely on calculator technology are not acceptable. Achieving a quadratic equation is sufficient working to then use a calculator to find the roots.

Question 3

Introduction | Question | Mark Scheme | Examiner Comments | Performance | Response A | Response B | Response C

Question 3 - Introduction

This was a short 3-mark question on two dimensional vectors. This was split into part (a) which was one mark requiring candidates to show how to find the magnitude of a vector, whilst part (b) required candidates to find the smallest value of *a* for which the magnitude of one vector was greater than another. Candidates who were unable to do part (a) were still able to do part (b).

Question 3 - Question

3. Relative to a fixed origin O

- the point A has position vector $5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
- the point B has position vector $2\mathbf{i} + 4\mathbf{j} + a\mathbf{k}$

where a is a positive integer.

(*a*) Show that $|\overrightarrow{OA}| = \sqrt{38}$

(1)

(*b*) Find the smallest value of a for which

$$|\overrightarrow{OB}| > |\overrightarrow{OA}|$$

(2)

(Total for Question 3 is 3 marks)

Question 3 - Mark Scheme

Question	Scheme	Marks	AOs
3(a)	$(\lvert\overrightarrow{OA}\rvert =)\ \sqrt{5^2+3^2+2^2} \ = \sqrt{38}$ *	B1*	1.1b
		(1)	
(b)	$\lvert\overrightarrow{OB}\rvert = \sqrt{2^2+4^2+a^2} = \sqrt{20+a^2}$ so when $a=5$ $\lvert\overrightarrow{OB}\rvert = \sqrt{20+25} = \sqrt{45}$	M1	1.1b
	$=5$	A1cso	2.3
		(2)	
		(3 marks)	

Question 3 - Examiner Comments

This was another short question on two-dimensional vectors with many candidates able to score 2 out of the 3 marks.

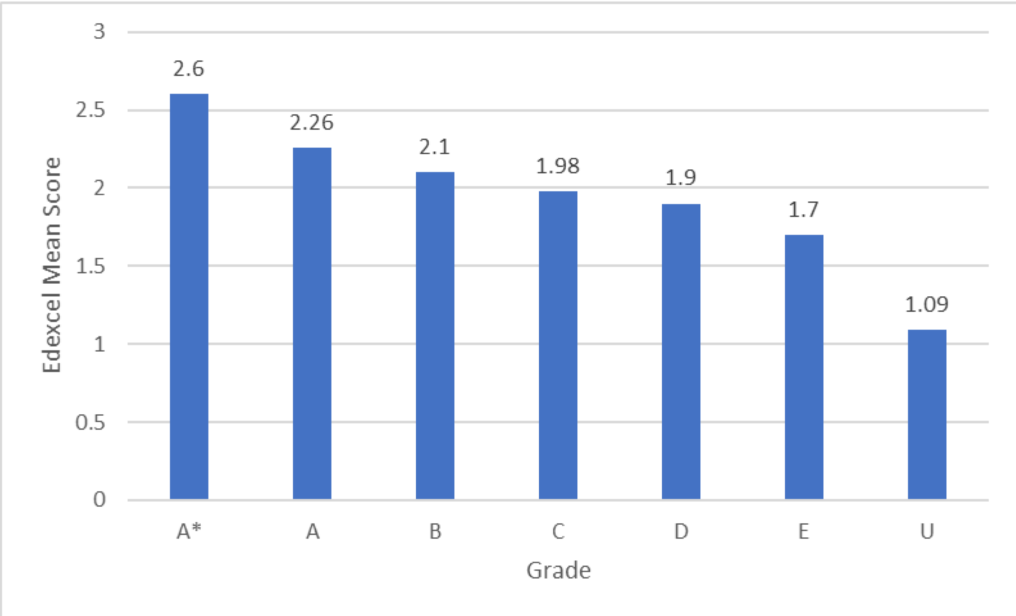
Part (a) was generally answered accurately by all candidates. Sometimes the mark was lost because the candidate omitted the square root and wrote $\lvert\overrightarrow{OA}\rvert = 25+9+4$. Sometimes the notation was not made clear or **i**, **j** and **k** appeared.

In part (b), the most common approach was to proceed to $a^2 > 18$ or $a^2 = 18$, and the majority of candidates were able to achieve this. However, a significant number of candidates did not conclude that $a=5$, with some claiming that e.g. $\sqrt{18}$ is an integer or missing the requirement that "a" had to be an integer. Some other incorrect answers included $a=\sqrt{19}$ following $a>\sqrt{18}$ and the answer $a \geq 5$ was not condoned, which lots the final mark for a number of candidates. A common processing error was the simplification of $\sqrt{20+a^2}$ to $\sqrt{20}+a$. Very few candidates used the substitution method and some of those did not reach any conclusion in the end.

Q3 i ? A B C

Question 3 - Performance

Mean score	Max score	Mean %	Edexcel averages: mean scored by candidates achieving grade: A*	A	B	C	D	E	U
2.12	3	71%	2.60	2.26	2.10	1.98	1.90	1.70	1.09



Q3
A
B
C

Question 3 - Response A

3) a) $A = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$ $O = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$ $\begin{matrix} -5 \\ -3 \\ -2 \end{matrix}$

$\sqrt{(-5)^2 + (-3)^2 + (-2)^2} = \sqrt{38}$

b) $|\overrightarrow{OA}| = \sqrt{38}$

$\underline{O}$ $\underline{D}$

$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 4 \\ a \end{pmatrix}$

$> \sqrt{38}$

$\begin{pmatrix} -2 \\ -4 \\ a \end{pmatrix}$

$(-2)^2 + (-4)^2 + (a)^2$

$4 + 16 + a^2$

$20 + (-4)^2:$

36

smallest value of A when

$|\overrightarrow{OB}| > |\overrightarrow{OA}|$ is

-5

1/3 marks

Q3 i ? A B C

Part (a)

B1*: They find the magnitude of AO which is acceptable. $\sqrt{(-5)^2+(-3)^2+(-2)^2}$ seen leading to $\sqrt{38}$ with no incorrect working seen.

Part (b)

M0: They form a correct equation in terms of a (seen across two lines), but do not proceed to e.g. $a^2 = "18"$, or substitute a positive value for a.

A0: Follows M0.

Examiner comments

The question stated that a was a positive integer so giving a negative answer suggests that the candidate had not read the question carefully.

Q3

i

?

A

B

C

Question 3 - Response B

$A\begin{pmatrix}5\\3\\2\end{pmatrix}$, $B\begin{pmatrix}2\\4\\a\end{pmatrix}$

a. $|\vec{OA}| = \sqrt{(5)^2+(3)^2+(2)^2}$

$\vec{OA} = \sqrt{38}$

b. $|\vec{OB}| = \sqrt{(2)^2+(4)^2+(a)^2}$ $= \cancel{2\sqrt{5} + \sqrt{(a)^2}}$

$\cancel{2\sqrt{5}}$

$\sqrt{20+(a)^2}$

$\vec{AB} = \vec{OB} + \vec{OA}$

$= \begin{pmatrix}2\\4\\a\end{pmatrix} + \begin{pmatrix}-5\\-3\\-2\end{pmatrix} = -3i + j + (a-2)$

$\sqrt{(3)^2+(1)^2+(a-2)^2}$

$\sqrt{38} + \sqrt{20+(a)^2} = \sqrt{10+(a-2)^2}$

$\sqrt{10+(a-2)^2}$

$38 + 20 + (a)^2 = \cancel{48/2}\ 10 + (a-2)^2$

$a^2 - 4a + 4$

$58 + a^2 = 10 + a^2 - 4a + 4$

$48 + \cancel{a^2} = \cancel{a^2} - 4a + 4$

$48 = -4a + 4$

$44 = -4a$

$a = -11$

a is positive

$\therefore a = 11$

$|\vec{OB}| = \sqrt{(2)^2+(4)^2+(11)^2}$

$= \sqrt{141}$ $= 11.87$

2/3 marks

Part (a)

B1*: $\sqrt{5^2+3^2+2^2}$ seen leading to $\sqrt{38}$ with no incorrect working seen. The lack of the modulus sign around OA on their second line is condoned.

Part (b)

M1: They make multiple unsuccessful attempts at setting up an equation. At the bottom of the page, they substitute a positive integer for a to find a value for the magnitude of OB, which scores this mark.

A0: Incorrect answer.

Question 3 - Response C

a) $5^2 + 3^2 + 2^2 = 38$

$\therefore \ |\overrightarrow{OA}| = \sqrt{38}$

b) $2^2 + 4^2 + 5^2 = 45$

$2^2 + 4^2 + \cancel{5}4^2 = 36$

$\therefore$ smallest value for a where $|\overrightarrow{OB}| > |\overrightarrow{OA}| = 5$

3/3 marks

Part (a)

B1*: $5^2+3^2+2^2=38 \Rightarrow |OA| = \sqrt{38}$ with no incorrect working seen.

Part (b)

M1: They substitute a positive integer for a to find a value for $|OB|^2$ (scored for either using $a=4$ or $a=5$)

A1: 5 cso

Examiner comments

Candidates typically set up an equation or an inequality in a, rather than just attempting values which this candidate did. This candidate had clearly appreciated the information given in the question which was that a was a positive integer.

Q3

A

B

C

Question 4

Question 4 - Introduction

This question was split into two parts: In part (a) candidates were required to use the small angle approximation for cos x to estimate the value of α, given the derivative of a function of a curve and that the small value of α was the x coordinate of the a stationary point of the curve. In part (b), candidates were expected to use the expression for the derivative in the question to find the equation of the tangent at the point $P(0,3)$

Question 4 - Question

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = \mathrm{f}(x)$ where $x \in \mathbb{R}$

Given that

- $\mathrm{f}'(x) = 2x + \frac{1}{2}\cos x$
- the curve has a stationary point with x coordinate α
- α is small

(*a*) use the small angle approximation for cos x to estimate the value of α to 3 decimal places.

(3)

The point $P(0, 3)$ lies on C

(*b*) Find the equation of the tangent to the curve at P, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(2)

(Total for Question 4 is 5 marks)

Question 4 - Mark Scheme

Question	Scheme	Marks	AOs
4a	$2\alpha+\frac{1}{2}\left(1-\frac{\alpha^2}{2}\right)$	M1	1.2
	$2\alpha+\frac{1}{2}\left(1-\frac{\alpha^2}{2}\right)=0 \Rightarrow 2\alpha+\frac{1}{2}-\frac{\alpha^2}{4}=0 \ \Rightarrow \alpha=\ldots$	dM1	1.1b
	$\alpha=-0.243$ (3dp) only	A1	2.3
		(3)	
b	$\mathrm{f}'(0)=\frac{1}{2}\cos 0 \Rightarrow \ldots \Rightarrow y=\ldots x+3$	M1	1.1b
	$y=\frac{1}{2}x+3$	A1	1.1b
		(2)	
		(5 marks)	

Question 4 - Examiner Comments

This question testing the small angle approximation for cosine and finding the equation of a straight line was accessible to many candidates with a large number scoring full marks.

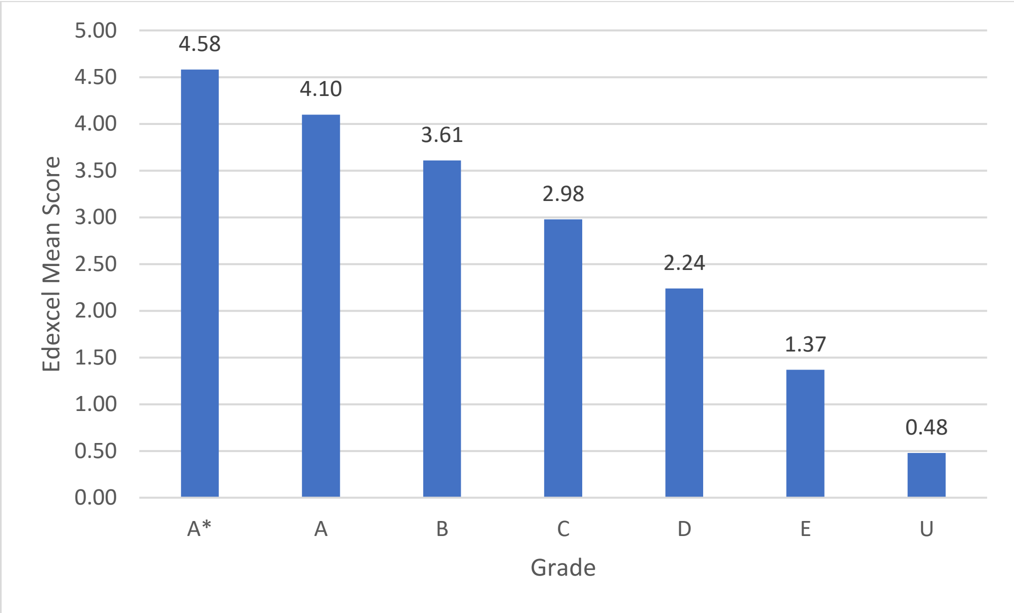
In part (a), some candidates formed and solved the equation but were not able to use the information that x was small to choose the correct solution. There was a very small minority of candidates who did not quote the correct small angle approximation, despite this being in the formula book. A slightly larger minority misunderstood that x being small meant that x was 0 and therefore did not include the $2x$ term in their solution attempt. Candidates should also pay closer attention to the question as several did not round to the required decimal places. It was also notable that some candidates at this level found it difficult to expand $\frac{1}{2}\left(1-\frac{x^2}{2}\right)$ when multiplying the fractions together.

Part (b) was mostly well answered with the majority scoring both marks, with most candidates gaining these marks efficiently without unnecessary calculations. Most candidates found the gradient of the tangent by substituting $x=0$ into the given derivative of $\mathrm{f}(x)$, however a small number of candidates took a longer approach by attempting to use the small angle approximation. Some candidates tried to substitute their answer from part (a) into the given derivative or found the gradient of the normal having originally found the correct gradient for tangent; these approaches lost both marks. A small number of candidates failed to realise the significance of the point (0,3) and therefore retained an algebraic gradient which they tried to substitute into $y=mx+c$. A number of responses were seen not giving their answer as an equation, just an expression $\frac{1}{2}x+3$, or forgetting to substitute their value of c found, giving their equation as $y=\frac{1}{2}x+c$; having been given the y intercept in the question, it was a requirement that for any marks to be scored that $c=3$.

Q4 i ? A B C

Question 4 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
3.32	5	66%	4.58	4.10	3.61	2.98	2.24	1.37	0.48



Question 4 - Response A

$f'(x) = 2\alpha + \frac{1}{2}\cos\alpha$

$= 2\alpha + \frac{1}{2}\left(1 - \frac{(\alpha)^2}{2}\right)$

$= 2\alpha + \frac{1}{2} - \frac{\alpha^2}{4}$

$= 4\alpha + 2 - \alpha^2$

$\alpha^2 - 4\alpha = 2$

$\alpha^2 - 4\alpha - 2 = 0$

$\alpha = \cancel{-0.449}, \ \alpha = 4.449$

Q4
A
B
C

b) $m = 2(4.449) + \frac{1}{2}\cot(4.449)$

$= 8.76$

$y - 3 = 8.76(x - 0)$

$y = 8.76x + 3$

2/5 marks

Part (a)

M1: Fully substitutes $\cos x = 1 - \frac{x^2}{2}$ into the derivative (seen on line 2 using α instead)

dM1: Attempts to multiply out to achieve a three-term quadratic (= 0) **and** attempts to find a value for α (the value 4.449 is correct for their quadratic – we needed to check this on a calculator to be able to award this mark)

A0: Incorrect

Part (b)

M0: Does not attempt to find the gradient of the curve when $x = 0$

A0: Follows M0

Examiner comments

Candidates were asked to show all stages of their working. Many solved their quadratic via a calculator without rearranging the equation which often resulted in errors when substituting in values for *a*, *b* and *c*

Question 4 - Response B

d). $f'(x) = 2x + \frac{1}{2}\cos x$

$= 2x + \frac{1}{2}\left(1 - \frac{x^2}{2}\right).$

$2x + \frac{1}{2} - \frac{1}{4}x^2 = 0.$

$\frac{1}{4}x^2 - 2x - \frac{1}{2} = 0.$

$\alpha = 4 \pm 3\sqrt{2}$

$\cancel{f'(x) = 2x + \frac{1}{2}\left(1 - \left(\frac{1}{2}\right)x^2\right)}$

$\cancel{= 2x + \frac{1}{2} - \frac{1}{8}x^2 = 0.}$

$\therefore \alpha = 4 - 3\sqrt{2}$

$= -0.243$

b). $f'(x) = 2x + \frac{1}{2}\cos(x).$

$= 2(3) + \frac{1}{2}\cos(0). = \frac{13}{2}$

$\therefore y = \frac{13}{2}x + 3$

3/5 marks

Part (a)

M1: Fully substitutes $\cos x = 1 - \frac{x^2}{2}$ into the derivative.

dM1: Attempts to multiply out to achieve a three-term quadratic (= 0) **and** attempts to find a value for α.

A1: -0.243 only cao

Part (b)

M0: Incorrect attempt to find the gradient of the curve when $x = 0$. They substitute $x = 3$ into part of the expression for $f'(x)$

A0: Follows M0

Q4

A

B

C

Question 4 - Response C

(2)

(a) $0 = 2x + \frac{1}{2}\left(1 - \frac{x^2}{2}\right)$

$0 = 2x + \frac{1}{2}\left(\frac{2-x^2}{2}\right)$

$0 = 2x + \frac{2-x^2}{4}$

$0 = \frac{8x+2-x^2}{4}$

$8x+2-x^2 = 0$

$x = 4+3\sqrt{2}$ $\qquad$ $x = 4-3\sqrt{2}$

$\boxed{x = 8.24}$

R.W

$\frac{1}{1} - \frac{x^2}{2}$

$\frac{2-x^2}{2}$

$\frac{2x}{1} \xrightarrow{\times 4} \frac{8x}{4}$

(b) $m = 2x + \frac{1}{2}\cos(x) \rightarrow y = mx + c \rightarrow y = mx + c$

$m = 2(0) + \frac{1}{2}\cos(0)$ $\qquad$ $3 = \frac{1}{2}(0) + c$ $\qquad$ $\boxed{y = \frac{1}{2}x + 3}$

$m = \frac{1}{2}$ $\qquad$ $c = 3$

4/5 marks

Part (a)

M1: Fully substitutes $\cos x = 1 - \frac{x^2}{2}$ into the derivative

dM1: Attempts to multiply out to achieve a three-term quadratic (= 0) **and** attempts to find a value for α. The use of x instead of α is condoned.

A0: Incorrect

Part (b)

M1: Attempts to find the gradient of the curve when $x = 0$ and achieves an equation of the form $y = "f'(0)"x + 3$

A1: $y = \frac{1}{2}x + 3$

Examiner comments

A significant number of candidates did not use the fact that α was small and that the small angle approximation can only be used under these circumstances. Many candidates selected 8.24 as their answer to part (a).

Question 5

Introduction | Question | Mark Scheme | Examiner Comments | Performance | Response A | Response B | Response C

Question 5 - Introduction

This question combined the trapezium rule with simultaneous equations. In part (a), candidates were asked to show that a given area was 17.59 using the given table of values. Part (b) required candidates to form another equation and solve simultaneously with the given equation in part (a).

Question 5 - Question

5. A continuous curve has equation $y = \mathrm{f}(x)$.

The table shows corresponding values of x and y for this curve, where a and b are constants.

x	3	3.2	3.4	3.6	3.8	4
y	a	16.8	b	20.2	18.7	13.5

The trapezium rule is used, with all the y values in the table, to find an approximate area under the curve between $x = 3$ and $x = 4$

Given that this area is 17.59

(*a*) show that $a + 2b = 51$

(3)

Given also that the sum of all the y values in the table is 97.2

(*b*) find the value of a and the value of b

(3)

(Total for Question 5 is 6 marks)

Question 5 - Mark Scheme

Question	Scheme	Marks	AOs
5(a)	$h = 0.2$	B1	1.1b
	$\frac{1}{2} \times "0.2" \times \{a + 13.5 + 2(16.8 + b + 20.2 + 18.7)\} = 17.59$	M1	1.1b
	e.g. $\Rightarrow a + 13.5 + 2b + 111.4 = 175.9 \Rightarrow a + 2b = 51$*	A1*	2.1
		(3)	
(b)	$a + 16.8 + b + 20.2 + 18.7 + 13.5 = 97.2 \Rightarrow a + b = 28 \Rightarrow a = ...$ (or $b = ...$)	M1	3.1a
	$a = 5$ or $b = 23$	A1	1.1b
	$a = 5$ and $b = 23$	A1	1.1b
		(3)	
			(6 marks)

Question 5 - Examiner Comments

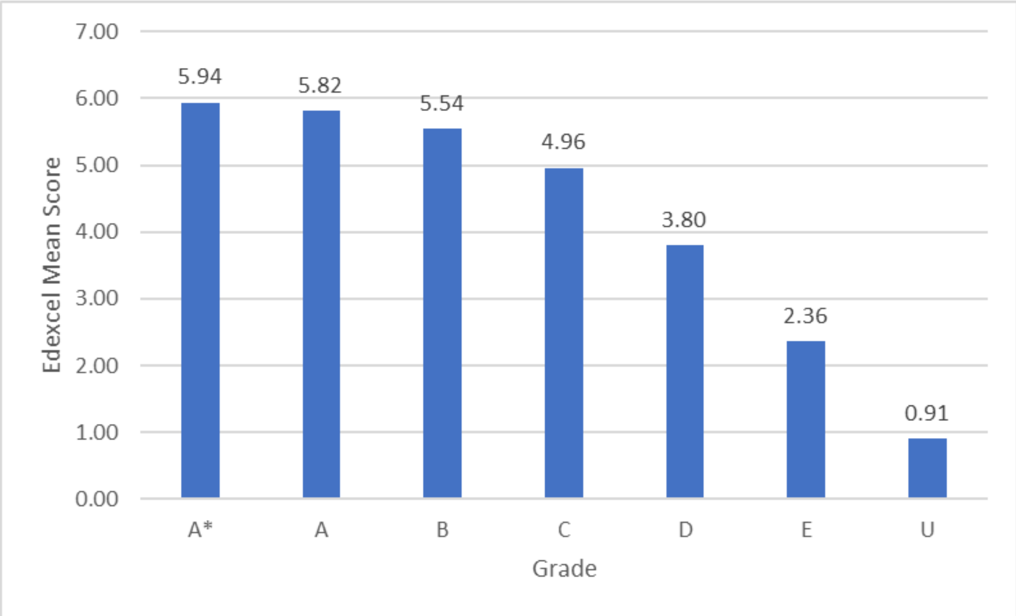
This was a lovely twist on a usually very standard question using the trapezium rule that was overall very well-answered. It was very rare to see a blank response to this question.

Candidates usually scored full marks in part (a). Where marks were lost the most common error was working out "h", the width of each strip, as $\frac{(4-3)}{6} = \frac{1}{6}$ where 6 was the number of ordinates given, rather than $\frac{(4-3)}{5} = \frac{1}{5}$, where 5 was the number of strips required. Very few candidates failed to give "h" a value. Most applied the standard trapezium rule correctly, although a minority failed to use the area of 17.59 to form an equation. Only a handful of solutions used the method of adding the areas of individual trapezia, which is much more time consuming and does not demonstrate as effectively an understanding the trapezium rule, which is provided in the formula booklet. The brackets were dealt with correctly in most cases although quite a number of candidates expanded the brackets before collecting terms, creating a more difficult equation to cope with. Others multiplied both sides by 10 which made the manipulation much easier. Several candidates had invisible brackets and recovered appropriately but lost the A mark as the answer was given. Although many values needed to be copied from the given table of x and y values, there were very few copying slips seen.

Part (b) was dealt with very well overall, with a variety of different methods used to solve the equations simultaneously, usually via the use of a calculator. Some arithmetic errors were seen in forming the second equation, $a + b = 28$, which resulted in the wrong values for a and b. Again, despite requiring the copying of many values from the given table of x and y values to form the required equation, there were few transcription errors. It is also worth noting that nearly all candidates used the given equation in (a) to correctly solve for values of a and b. Although it was made possible for them to gain the method mark for using "their" equation, this was very infrequently utilised, or required. Furthermore, it was pleasing to see candidates who made no attempt at (a) still proceeded to use the given answer to attempt and generally gain full marks in (b).

Question 5 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	Max score	Mean %	A*	A	B	C	D	E	U
4.93	6	82%	5.94	5.82	5.54	4.96	3.80	2.36	0.91



Question 5 - Response A

difference = 0.2

a) $\left(\frac{a}{2} + 16.8 + 6 + 20.2 + 18.7 + \cancel{\frac{13.5}{2}}\ 6.75\right)0.2$

$= 17.55$

$\frac{17.55}{0.2} = \cancel{8.795}\ 87.95 = \cancel{(0.5a + 16.8 + 6 + 20.2}$

$\cancel{8.795} = (0.5a + 16.8 + 6 + 20.2 + 18.7 + 6.75)$

$87.95 = (0.5a + 26 + 62.45)$

$\cancel{8.795 - 16.8 - 20.2 - 18.7 - 6.75 =}$

$87.95 - 62.48 =$

$\cancel{87.95 - 62.45} = 25.5 = \frac{1}{2}a + 26$

$\times 2 \quad \frac{a}{2} + 26 = 25.5 \quad) \times 2$

$a + 26 = 51$

b) $97.5 - 62.45 = 35.05$

$\cancel{\frac{1}{2}a + 26} = 35.05$

$2a + 26 = 70.1$

$\cancel{2a + 2}$

$\frac{1}{2}a + 26 = 35.05$

$a + 26 = 70.1$

$a : 6$

$1 : 2 = \cancel{70.1}\ 3$

$23.4 : 46.7 = 70.1$

$a = 23.37$

$b = 46.73$

2/6 marks

Part (a)

B1: $h = 0.2$ implied.

M1: Forms the equation applying the trapezium rule with their h. This is an equivalent version where all the terms have been halved so the $\frac{1}{2}$ is not outside the bracket.

A0*: An error occurs on line 6 of their working (b becomes $2b$).

Part (b)

M0: They do not attempt to sum the y values together to form a second equation. They instead use the sum of the values from the trapezium rule formula in (a).

A0A0: Follows M0

Examiner comments

The solution to part (a) was difficult to follow with lots of crossing out and manipulating numbers into other numbers. Candidates should make sure that their solutions to "show that" questions are clear and free from errors and/or omissions of brackets.

Question 5 - Response B

a) $17.59 = \frac{1}{2}\left(\frac{9-3}{6}\right)\times\left(\left(a+13.5\right)+2\left(16.8+b+20.2+18.7\right)\right)$

$211.08 = a + 13.5 + 111.4 + 2b$

$211.08 - 13.5 - 111.4 = a + 2b$

$a + 2b = 51$

b)

$97.2 - 69.2 = a + b$

$a + b = 28$

$a = 28 - b$

sub into $a + 2b = 51$

$28 - b + 2b = 51$

$28 + b = 51$

$b = 23$

$a = 5$

4/6 marks

Part (a)

B0: Incorrect h

M1: Forms the equation applying the trapezium rule with their h

A0*: Follows B0

Part (b)

M1: The equation $a+b=28$ is formed and proceeds to find a value for a or b

A1: $a=5$ or $b=23$

A1: $a=5$ and $b=23$

Q5 A B C

Examiner comments

Most candidates were able to score full marks in part (b). It was acceptable to solve the two simultaneous equations directly via a calculator and sight of a value for *a* or *b* following two simultaneous equations was sufficient to score the method mark.

Question 5 - Response C

a) $h = 0.2$

trapezium rule $= \frac{1}{2}h\left((y_0 + y_n) + 2(y_1 + y_2 + \dots y_{n-1})\right)$

$17.59 = \frac{1}{2}(0.2)\left((a + 13.5) + 2(16.8 + b + 20.2 + 18.7\right.$

$17.59 = \frac{1}{10}\left((a + 13.5) + 2(55.7 + b)\right.$

$17.59 = \frac{1}{10}\left((a + 13.5) + 111.4 + 2b\right.$

$17.59 = \frac{1}{10}(a + 13.5 + 111.4 + 2b)$

$17.59 = \frac{1}{10}(a + 124.9 + 2b)$

$17.59 = \frac{1}{10}a + 12.49 + 2b$

$5.1 = \frac{1}{10}a + 2b$

$\therefore \; 51 = a + 2b \qquad a + 2b = 51$

b) $a + 16.8 + b + 20.2 + 18.7 + 13.5 = 97.2$

$69.2 + a + b = 97.2$

$~~51 = a + 2b~~$

$~~51 - 2b = a~~$

$~~(51 - 2b) + b = 69.2~~$

$~~51 - 2b + b = 69.2~~$

$~~51 - b = 69.2~~$

$~~51 - 69.2 = b~~$

$~~51 - 2b + b = 69.2~~$

$51 = a + 2b$

$51 - 2b = a$

$69.2 + a + b = 97.2$

$a + b = 28$

$(51 - 2b) + b = 28$

$51 - 2b + b = 28$

$51 - b = 28$

$51 - 28 = b$

$b = 23$

$51 = a + 2(23)$

$51 = a + 46$

$5 = a$

$\therefore a = 5$

$b = 23$

5/6 marks

Part (a)

B1: $h = 0.2$ seen

M1: Forms the equation applying the trapezium rule with their *h*; we condone the missing trailing bracket.

A0: Incorrect working seen (in the penultimate two lines). The missing trailing bracket would have been condoned for this mark had there been no other errors.

Part (b)

M1: The equation $a + b = 28$ is formed and proceeds to find a value for *a* or *b*

A1: $a = 5$ or $b = 23$

A1: $a = 5$ and $b = 23$

Q5

i ? A B C

Question 6

Introduction | Question | Mark Scheme | Examiner Comments | Performance | Response A | Response B | Response C

Question 6 - Introduction

This question tested logarithms. It was split into three separate parts which were all independent but increasing in demand.

Question 6 - Question

6. $a = \log_2 x \qquad b = \log_2(x + 8)$

Express in terms of a and/or b

(*a*) $\log_2 \sqrt{x}$

(1)

(*b*) $\log_2(x^2 + 8x)$

(2)

(*c*) $\log_2 \left(8 + \frac{64}{x}\right)$

Give your answer in simplest form.

(3)

(Total for Question 6 is 6 marks)

Question 6 - Mark Scheme

Question	Scheme	Marks	AOs
6(a)	$\frac{1}{2}a$	B1	2.2a
		(1)	
(b)	$\log_2 x(x+8) \Rightarrow \log_2 x + \log_2(x+8)$	M1	1.2
	$= a+b$	A1	2.2a
		(2)	
(c)	e.g. $8+\frac{64}{x}=\frac{8x+64}{x}$	B1	1.1b
	$\log_2 \frac{8}{x}(x+8) = 3-\log_2 x+\log_2(x+8)$	M1	1.1b
	$3+b-a$	A1	2.2a
		(3)	
		(6 marks)	

Question 6 - Examiner Comments

This was again another accessible question with a total of 6 marks split between 3 different sub-parts. However, this topic continues to be one which candidates struggle with and in all parts of this question. It was not uncommon to see candidates attempting to use rules of indices rather than logarithms, starting off with $2^a = x$, $2^b = x+8$ and attempting (usually incorrectly) to substitute these into the expressions given. Where this was successful candidates generally failed to remove $\log_2 2$ resulting in them losing one mark per part.

Part (a) was a B mark for achieving the correct answer. Several candidates got as far as $0.5\log_2 x$ without substituting *a* into this and so gained no credit. Others could not apply the index log law to the square root of *x* correctly and so ended up with the answer square root of *a* often written as $a^{\frac{1}{2}}$

Part (b) surprisingly seemed to have more success. It was generally answered well with most candidates realising that factorising the argument of the logarithm was needed before applying the addition law of logarithms. There were several candidates who, having factorised correctly, applied the addition law incorrectly, resulting in an answer of $a \times b$ and so lost both marks. Several candidates just wrote down the correct answer and gained both marks. If the correct answer followed the product of the relevant logs, $\log(x)\times\log(x+8)$ rather than the sum, then the answer was allowed to imply the correct method, but the final mark was withheld for this incorrect working seen. The main misconception was the sight of $\log x^2 + \log 8x$ which was awarded no marks, although writing $\log x + \log + \log x + \log 8$ (possibly seen as $2a+3$) gained the method mark as evidence of correct

Q6

A

B

C

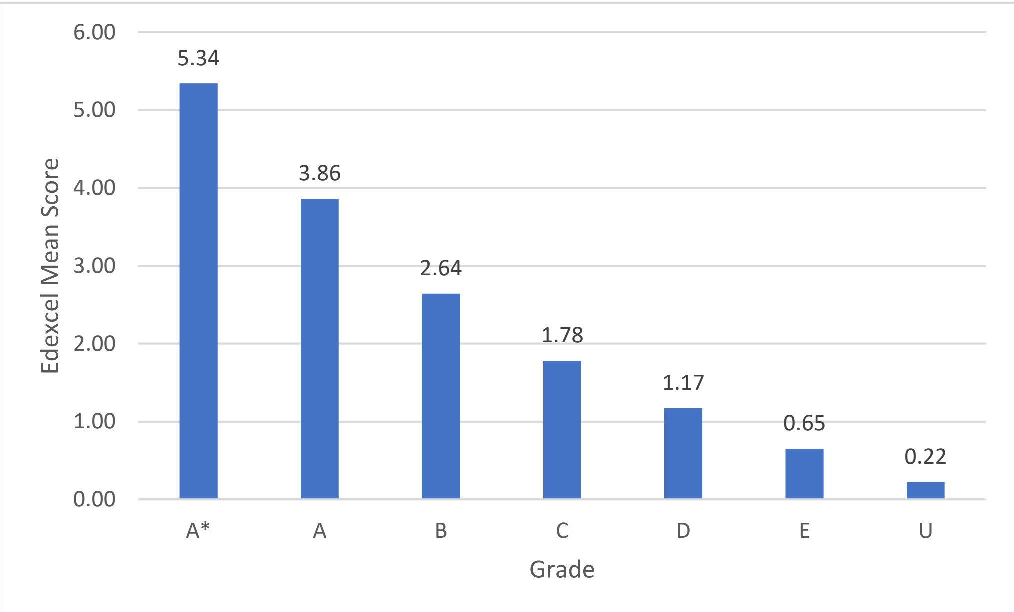
addition law used at some point. Some candidates attempted to work backwards from the given log expressions for *a* and *b* with very limited success.

Part (c) was the most challenging part of this question and clearly identified the stronger mathematicians of the cohort. Many responses to this part were blank, whilst the more able candidates provided very succinct solutions. For those who attempted with less success there were some rather dubious attempts at writing $8+\frac{64}{x}$ as a single fraction. Of those who managed this correctly and progressed to applying the laws of logarithms, quite a few left in the $\log_2 8$ rather than simplifying to its value of 3, which meant that only the first mark was available: marks had already been awarded for the power law and addition law in previous parts of the question and here we required an understanding of reaching an answer in its simplest form. This further demonstrated the lack of logarithmic fluency for a number of candidates. Others factorised 8 from $8+\frac{64}{x}$ but then struggled to apply the laws appropriately as they had not completed the process of writing the argument as a single fraction.

Misconceptions were seen such as expressing $\log\left(8+\frac{64}{x}\right)$ as $\log 8+\log\left(\frac{64}{x}\right)$ and sign errors often occurred.

Question 6 - Performance

Mean score	Max score	Mean %	Edexcel averages: mean scored by candidates achieving grade: A*	A	B	C	D	E	U
2.83	6	47%	5.34	3.86	2.64	1.78	1.17	0.65	0.22



Question 6 - Response A

?

(a) $a^{1/2}$

(b) $\log_2(x+8) + \log_2(x) = \log(x^2+8x) \Rightarrow ab$

(c) $\log_2(x+8) + \log_2(x+8) = \cancel{\log_2(x^2+8x+64)}$ $\frac{2b}{a}$

1/6 marks

Part (a)

B0: Incorrect

Part (b)

M1: Takes a factor of x out of the bracket to achieve $\log_2 x(x+8)$ and attempts to apply the addition law of logarithms, leading to $\log_2 x + \log_2(x+8)$

A0: ab is incorrect.

Part (c)

B0: Does not write $8+\dfrac{64}{x}$ as a single fraction.

M0: Does not proceed to $3 \pm \log_2 x \pm \log_2(x+8)$ and this is not implied by $\dfrac{2b}{a}$

A0: Follows

Examiner comments

In part (b) the candidate appears to try and work to the given expression in the question, which was an acceptable approach, but solutions were sometimes quite difficult to follow.

Q6

A
B
C

Question 6 - Response B

a) $\log_2 \sqrt{x} = \log_2 x^{\frac{1}{2}} = \frac{1}{2}\log_2 x = \frac{1}{2}a$ //

b) $\log_2 (x^2+8x) = \log_2 (x+8)(x) = \log_2 (x+8) + \log_2 x$

$= b + a$ //

c) $\log_2 \left(8 + \frac{64}{x}\right) = \log_2 (8) + \log_2 \left(\frac{64}{x}\right)$

$= 3 + \log_2 (64) - \log_2 (x)$

$= 3 + 6 - \log_2 (x)$

$= 9 - a$ //

3/6 marks

Part (a)

B1: $\frac{1}{2}a$

Part (b)

M1: Takes a factor of x out of the bracket to achieve $\log_2 x(x+8)$ and attempts to apply the addition law of logarithms (scored on line 1 of (b))

A1: $a+b$ or simplified equivalent ($b+a$ is fine)

Part (c)

B0: Does not write $8+\frac{64}{x}$ as a single fraction.

M0: Does not proceed to $3 \pm \log_2 x \pm \log_2 (x+8)$

A0: Follows

Examiner comments

The approach in part (c) to split up the logarithm incorrectly was a common misconception.

Q6

A

B

C

Question 6 - Response C

a) $\log_2 x^{1/2}$

$\frac{1}{2}\log_2 x$

$\frac{1}{2}a$

b) $\log_2(x^2+8x)$

$\log_2 x(x^{\cancel{2}}+8\cancel{x})$

$= \log_2 x + \log_2(x+8)$

$= a + b$

c) $\log_2\left(8+\frac{64}{x}\right)$

$= \log_2 8\left(1+\frac{8}{x}\right)$

$= \log_2 8 + \log_2\left(1+\frac{8}{x}\right)$

$= \log_2 8 + \log_2\left(\frac{x+8}{x}\right)$

$= \log_2 8 + (\log_2(x+8) - \log_2 x)$

$= \log_2 8 + a - b$

$= \cancel{3+a+b}$

$= 3 + a - b$

5/6 marks

Part (a)

B1: $\frac{1}{2}a$ (line 3)

Part (b)

M1: Takes a factor of x out of the bracket to achieve $\log_2 x(x+8)$ and attempts to apply the addition law of logarithms (line 3 of (b))

A1: $a+b$

Part (c)

B1: Writes $8+\frac{64}{x}$ as a single fraction (implied by line 4)

M1: Attempts to apply the laws of logs, uses $\log_2 8=3$ and proceeds to $3\pm\log_2 x\pm\log_2(x+8)$ or equivalent.

A0: Incorrect

Examiner comments

Candidates had to deal with $\log_2 8=3$ in part (c) because the other laws of logarithms had already been tested in the earlier parts.

Question 7

Introduction | Question | Mark Scheme | Examiner Comments
Performance | Response A | Response B | Response C

Question 7 - Introduction

This question tested the topic of functions. In part (a), candidates were required to demonstrate their understanding of the term "range", in part (b) candidates were required to find the inverse function of f, part (c) tested knowledge of composite functions, whilst part (d) asked candidates to find the exact value of a constant *a* which satisfied an equation involving both functions.

Question 7 - Question

7. The function f is defined by

$$\mathrm{f}(x) = 3 + \sqrt{x-2} \qquad x \in \mathbb{R} \qquad x > 2$$

(*a*) State the range of f

(1)

(*b*) Find f^{-1}

(3)

The function g is defined by

$$\mathrm{g}(x) = \frac{15}{x-3} \qquad x \in \mathbb{R} \qquad x \neq 3$$

(*c*) Find gf(6)

(2)

(*d*) Find the exact value of the constant *a* for which

$$\mathrm{f}(a^2 + 2) = \mathrm{g}(a)$$

(2)

(Total for Question 7 is 8 marks)

Question 7 - Mark Scheme

Question	Scheme	Marks	AOs
7(a)	$f(x) > 3$	B1	1.1b
		(1)	
(b)	$y = 3+\sqrt{x-2} \Rightarrow x = ...$	M1	1.1b
	$f^{-1}(x) = (x-3)^2 + 2$	A1	1.1b
	$x > 3$	B1ft	2.2a
		(3)	
(c)	$f(6) = 3+\sqrt{6-2} = 5 \Rightarrow g("5") = \frac{15}{"5"-3} = ...$	M1	1.1b
	$= \frac{15}{2}$	A1	1.1b
		(2)	
(d)	$3+\sqrt{a^2+2-2} = \frac{15}{a-3} \Rightarrow "a^2 - 9 = 15"$	M1	1.1b
	$a = 2\sqrt{6}$	A1	2.2a
		(2)	
			(8 marks)

Question 7 - Examiner Comments

Overall, this was an accessible question on functions which allowed candidates to attempt all sections of it even when they found parts (a) or (b) particularly challenging. Part (c) had the lowest number of errors out of the entire question and was one of the most successfully answered parts over the entire paper.

In part (a), a large majority of candidates were able to score the B mark available, with nearly all attempting to give the range. Although the mark scheme was generously accepting notations such as $f > 3$ or range > 3, some candidates lost the mark for unacceptable labels such as $x > 3$ or $f(x) \geq 3$.

In general, candidates correctly rearranged the formula and interchanged *x* and *y* successfully in part (b). A small number of candidates made some errors in applying the correct order of operations and this resulted in an incorrect expression for the inverse function. A common error was seen with candidates who incorrectly manipulated $y = 3+\sqrt{x-2}$ to obtain $y^2 = 9 + x - 2$. The candidates who lost the A mark were often due to labelling the inverse as $y =$ instead of e.g. $f^{-1}(x) = ...$ A small minority of candidates misread $f^{-1}(x)$ as $f'(x)$ and so they differentiated the function instead of finding the inverse of it. Candidates should be advised to read the question and the labels carefully to check which skill is required. A common error costing candidates the B mark was failing to record the domain of the inverse function. This mark was rarely scored across the entire cohort, despite 3 marks possibly indicating that more was required by candidates than just rearranging to find $f^{-1}(x)$. Candidates who paid careful attention to how f and g were defined in the question would be able to attempt similarly for the inverse function.

Part (c) was well attempted by many candidates even when they could not attempt parts (a) or (b). Most candidates scored both marks available in this part. Finding $f(6)$ first and then substituting this result into g was more common than substituting $x = 6$ into the composite function gf(x). This second approach caused some errors due to incorrect algebraic manipulation leading to various incorrect answers for the composite function.

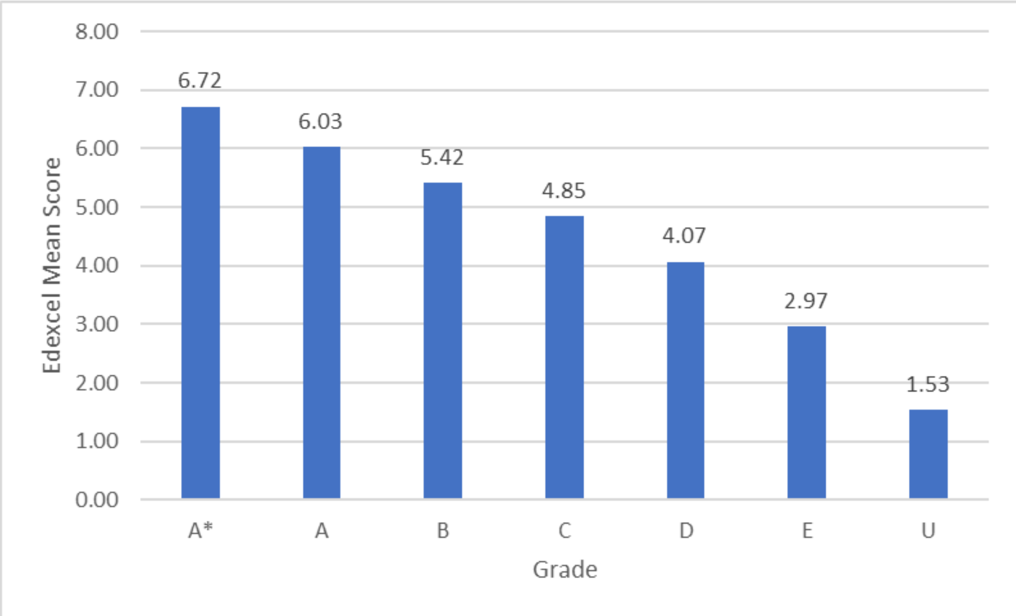
A large number of candidates scored the first mark in part (d) as they correctly formed the equation $(a-3)(3+a) = 15$ and proceeded to a quadratic in a. Several candidates failed to spot that $\sqrt{a^2} = a$, or that $a^2 + 2 - 2 = a^2$. Some candidates that found $a = \pm 2\sqrt{6}$ did not reject the negative solution losing the last A mark. Some candidates possibly rejected the negative solution by seeing that the domain of f had to be greater than 2 and fortuitously scored the final mark. However, others may have considered that the $\left(-2\sqrt{6}\right)^2 + 2$ also satisfied this domain resulting in $f\left(\left(-2\sqrt{6}\right)^2 + 2\right) > 0$, but that $g\left(-2\sqrt{6}\right) < 0$ so this negative solution had to be rejected. Other errors included writing decimal answers only. On several occasions $a^2 - 9 = 15$ incorrectly became $a^2 = 6$.

A number of candidates were not able to obtain the quadratic equation needed (hence, did not score any marks) because they got confused by $3 + \sqrt{a^2}$ and not realising that this was actually $3 + a$. Others tried to manipulate the equation leading to a quartic equation and usually solved via a calculator (and successfully in a few cases), however those who had proceeded along this route typically were expected to apply algebraic division or the factor theorem to find the quadratic factor, having introduced other solutions from squaring and the complexity of their expressions meant it was rare for the method mark to be scored.

Q7
i
?
A
B
C

Question 7 - Performance

Mean score	Max score	Mean %	Edexcel averages: mean scored by candidates achieving grade: A*	A	B	C	D	E	U
5.19	8	65%	6.72	6.03	5.42	4.85	4.07	2.97	1.53



Question 7 - Response A

(2)

a) $f(x) = 3 + \sqrt{2-2} = 3$

$f^{-1} = \quad y = 3 + \sqrt{x-2}$

$y - 3 = \sqrt{x-2}$

$y^2 - 3^2 = x - 2$

$y^2 - 9 = x - 2$

$y^2 - 7 = x$

$f^{-1} = y^2 - 7$

c) ~~gf(x)~~ $gf(6)$

$f(x) = 3 + \sqrt{6-2}$
$= 5$

$g(5) = \frac{15}{5-3} = \frac{15}{2} = 7.5$

d) $f(a^2+2) = g(a)$

$(3 + \sqrt{x-2} \; + 2)$

2/8 marks

Q7

A

B

C

Part (a)

B0: $f(x) > 3$ or equivalent is not seen.

Part (b)

M0: Although not labelled, the work under (a) appears to be (b). They do not achieve the required form to score this mark, however.

A0: Follows M0.

B0ft: No attempt.

Part (c)

M1: Substitutes $x = 6$ into f and substitutes the result into g to find a value for $gf(6)$.

A1: $\frac{15}{2}$ (or 7.5)

Part (d)

M0: No valid attempt to form the required equation.

A0: Follows M0.

Examiner comments

Part (c) was typically correct for most candidates even if they had little success with the other parts of the question.

Question 7 - Response B

a) $f(x) = 3 + \sqrt{x-2}$

$y > 3$

b) $3 + \sqrt{x-2} = y$

$\sqrt{x-2} = y - 3$

$x - 2 = \sqrt{y^2 - 3}$

$x = 2 + \sqrt{y^2 - 3}$

$f^{-1}(x) = 2 + \sqrt{x^2 - 3}$

c) $f(6) = 3 + \sqrt{6-2}$

$= 3 + 2$

$= 5$

$g(5) = \frac{15}{5-3} = \frac{15}{2} = 17,5$

d) $f(a^2+2) = g(a)$

$3 + \sqrt{a^2+2-2} = \frac{5}{a-3}$

$3 + a = \frac{5}{a-3}$

$(3+a)(a-3) = 5$

$3a - 9 + a^2 - 3 = 5$

$a^2 - 9 = 5$

$a^2 = 14$

$a = \pm\sqrt{14}$

$a = \sqrt{14}$ as $x > 2$

4/8 marks

Part (a)

B1: $y > 3$

Part (b)

M0: They attempt to rearrange the equation, but do not proceed to an expression for x of the required form.

A0: Follows M0.

B0ft: No attempt to state the domain $x > 3$

Part (c)

M1: Substitutes $x = 6$ into f and substitutes the result into g to find a value for $gf(6)$

A1: $\frac{15}{2}$ (isw the 17.5)

Part (d)

M1: They attempt to form the required equation and proceed to a quadratic in a. (the miscopying of 15 written as 5 on the right-hand side and -3 instead of -3a when they multiply out the brackets are condoned)

A0: Incorrect

Examiner comments

The notation used in (a) was condoned on this paper, although candidates should be encouraged to use e.g. $f(x) > 3$ on similar questions in the future.

Question 7 - Response C

(2)

a) $f(x) > 3$ $\quad\left(\text{as } f(x) = 3 + \sqrt{2-2} = 3\right)$

b) $x = 3 + \sqrt{y-2}$

$x - 3 = \sqrt{y-2}$

$(x-3)^2 = y - 2$

$(x-3)^2 + 2 = y$

$f^{-1}(x) = (x-3)^2 + 2$

$x - 3 = \sqrt{y-2}$

$(x-3)^2 = y - 2$

$(x-3)^2 + 2 = y$

$\boxed{y = (x-3)^2 + 2, \; x > 3}$

c) $f(6) = 3 + \sqrt{6-2}$

$= 3 + \sqrt{4}$

$= 5$

$g(5) = \frac{15}{5-3} = \frac{15}{2}$ //

d) $f(a^2+2) = 3 + \sqrt{a^2}$
$= 3 + a$

$g(a) = \frac{15}{a-3}$

$3+a = \frac{15}{a-3}$

$a^2 - 9 = 15$

$a^2 = 24$

$a = 2\sqrt{6}$ //

verify : $\frac{15}{2\sqrt{6}-3} = 3+2\sqrt{6}$ g ✓

$3 + \sqrt{(2\sqrt{6})^2 - 2} = 3+2\sqrt{6}$ f ✓

7/8 marks

Part (a)

B1: $f(x) > 3$ seen.

Part (b)

M1: Sets $x = 3 + \sqrt{y-2}$ and attempts to make y the subject achieving an expression of the required form.

A0: They have not used allowable notation for the function in this part.

B1ft: $x > 3$ seen.

Part (c)

M1: Substitutes $x = 6$ into f and substitutes the result into g to find a value for $gf(6)$

A1: $\frac{15}{2}$

Part (d)

M1: Forms the required equation and proceeds to a quadratic in a

A1: $(a =)\ 2\sqrt{6}$ is correct.

Examiner comments

It was rare for candidates to give the domain in part (b).

Question 8

Question 8 - Introduction

This question tested knowledge of radians and the application of this angle measure to solve a problem in context. In parts (a) and (b), candidates were required to demonstrate knowledge of the formula for the length of an arc and understanding of the sum of the angles in a straight line in radians. In part (c), candidates were able to use the given facts to help them to find the total area of the stage, which required use of the formula for the area of the sector, as well as the formula for the area of a triangle.

Question 8 - Question

8.

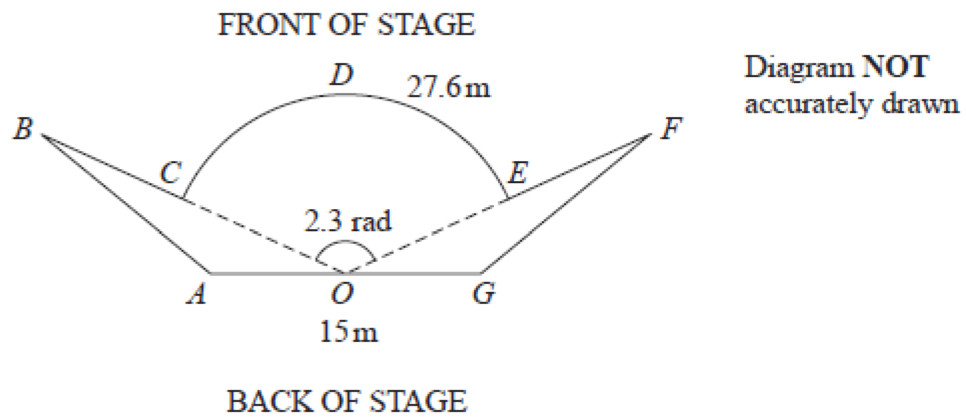


Figure 1

Figure 1 shows the plan view of a stage.

The plan view shows two congruent triangles *ABO* and *GFO* joined to a sector *OCDEO* of a circle, centre *O*, where

- angle *COE* = 2.3 radians
- arc length *CDE* = 27.6 m
- *AOG* is a straight line of length 15 m

(*a*) Show that *OC* = 12 m.

(2)

(*b*) Show that the size of angle *AOB* is 0.421 radians correct to 3 decimal places.

(2)

Given that the total length of the front of the stage, *BCDEF*, is 35 m,

(*c*) find the total area of the stage, giving your answer to the nearest square metre.

(6)

(Total for Question 8 is 10 marks)

Question 8 - Mark Scheme

Question	Scheme	Marks	AOs
8(a)	$OC \times 2.3 = 27.6$	M1	1.1b
	e.g. $OC = \dfrac{27.6}{2.3} = 12\text{ m}$ *	A1*	2.1
		(2)	
(b)	e.g. $(2AOB =)\ \pi - 2.3$	M1	1.1b
	$\dfrac{\pi - 2.3}{2} \Rightarrow 0.421\ \text{rad}$ *	A1*	2.1
		(2)	
(c)	Area $OCDE = \dfrac{1}{2} \times 12^2 \times 2.3$	M1	1.1b
	$= 165.6\ (\text{m}^2)$ (accept awrt 166)	A1	1.1b
	$(OB =)\ \dfrac{35 - 27.6}{2} + 12 = 15.7\text{ m}$	B1	2.1
	Area of OAB (or OFG) $= \dfrac{1}{2} \times "15.7" \times 7.5 \times \sin 0.421\ \ (= 24.0...\text{m}^2)$	M1	1.1b
	Total area $= "165.6" + 2 \times "24.1"$	dM1	3.1a
	$=$ awrt $214\ (\text{m}^2)$	A1	1.1b
		(6)	
			(10 marks)

Q8

i ? A B C

Question 8 - Examiner Comments

Generally most candidates found this question accessible, working confidently in radians throughout, although inadvisable conversions to degrees were not uncommon with many candidates unaccountably preferring the unwieldy $\frac{\theta}{360}\times\pi r^2$ to the elegant $\frac{1}{2}r^2\theta$.

In part (a) a rigorous solution was expected; a very large number of candidates lost the second mark by failing to explicitly show the process of dividing the arc length by the angle (perhaps the mark most commonly lost on this question). Candidates need to appreciate that every step of working needs to be clearly demonstrated in order to gain full marks for "show that" questions and that their final result needs to be what the question asked them to show in the first place. As the question was worth 2 marks, candidates should not expect to just state $2.3r = 27.6$ and for this to be sufficient. Those who chose to convert 2.3 radians to degrees not only lost time but also invariably lost the accuracy mark. Others did show the division and did not label their answer so just an expression was seen.

Part (b) was typically answered well, with many candidates recognising the straight line giving a total of π radians and allowing them to solve the problem. Where candidates lost marks, this was often due to a lack of brackets, e.g. $\pi - 2.3 \div 2 = 0.421$, or an incorrect joined statement such as $\pi - 2.3 = \frac{0.842...}{2} = 0.421$. A small minority worked in degrees which needed much more work to secure both marks.

There was a wide range of marks for part (c), although many candidates produced highly competent solutions and gained full marks. Most candidates seemed familiar with the area of a sector formula and the vast majority scored both marks for this. Candidates also seemed fully aware of the area of a triangle formula, although they did not always use a correct method to work out the length 15.7. Some candidates seemed not to realise that they needed to use the information provided, i.e. the length of the front of the stage, in order to obtain all the lengths needed for their area calculations. There were also some more complex and longer methods to find the area of a triangle which were not generally successful, but the most common error was using 12 as opposed to 15.7. Some candidates spent time working out the length of *AB* but then realised that this was not necessary.

Q8

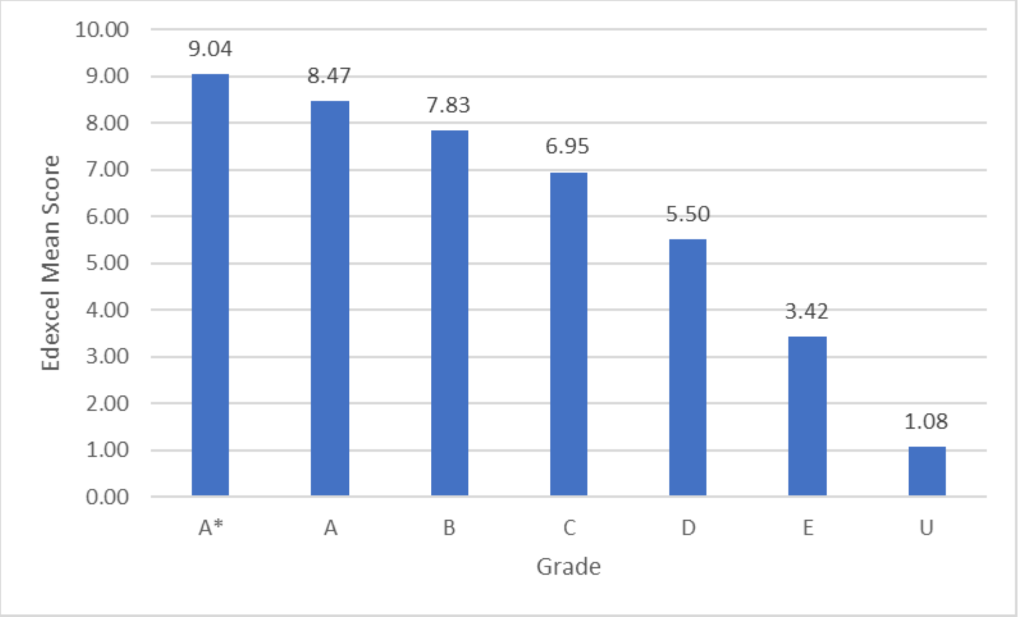
A

B

C

Question 8 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	Max score	Mean %	A*	A	B	C	D	E	U
7.17	10	72%	9.04	8.47	7.83	6.95	5.50	3.42	1.08



Q8
A
B
C

Question 8 - Response A

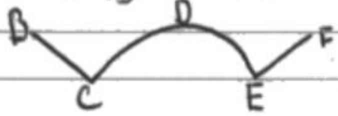
arc L = $r \times \theta$

a) $27.6 = r\,2.3$

$r = 12$ OC = r
OC = 12

b) Aob $= \frac{\pi - 2.3}{2} = 0.421$

c) BCDEF = 35



$35 - 27.6 = 7.4$

$\frac{7.4}{2} = Bc \Rightarrow 3.7$

$\frac{1}{2}\pi\theta$ = area of arc

= ~~360~~ 258.

2.3
3.49

~~180~~ 57.79

OCDE = ~~[illegible]~~ 87.1m 48π

Bc + OC = ~~[illegible]~~. $15.7 = H$

$\frac{1}{2} \times 15.7 \times$ ~~[illegible]~~ $\frac{15}{2}$

AOB = 58.875
58.9

A = 205m^2

4/10 marks

Q8 i ? A B C

Part (a)

M1: Uses $l = r\theta$ with $l = 27.6$ and $\theta = 2.3$ correctly substituted in ($27.6 = r2.3$ scores this mark)

A0*: Does not rearrange the equation to achieve an expression for OC

Part (b)

M1: Attempts to subtract 2.3 from π

A1*: Achieves 0.421 (rad) with no errors seen, following a correct expression for angle AOB

Part (c)

M0: Does not use $A = \frac{1}{2}r^2\theta$ with $r = 12$ and $\theta = 2.3$

A0: Follows M0

B1: 15.7 seen

M0: Incorrect attempt to find the area of at least one of the two congruent triangles.

dM0A0: Follows M0

Examiner comments

Many candidates lost the final mark in part (a) for not rearranging their equation and concluding with what they had been asked to show.

Q8

i

?

A

B

C

Question 8 - Response B

a) $l = r\theta$

$27.6 = r\,2.3$

$\frac{27.6}{2.3} = 12$ $OC = 12$

b) AOG Straight line $\frac{\pi - 2.3}{2} = 0.4207$

$\pi = 2.3\text{rad} + AOC + \angle GOE$ 0.421 rad

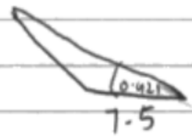
$\pi = 2.3\text{rad} + \text{angle } AOC + \text{Angle } GOE$ Angle $AOB = 0.421$ rad

8c) $BCDEF = 35m$

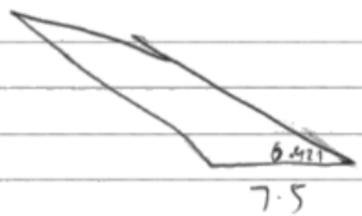
$AG = 15$

Total area = $BCDEF + AG + AB + GF$

$35 + 15$

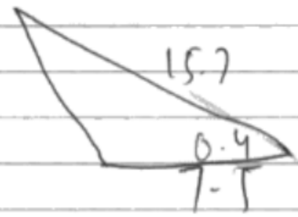


$\frac{2.3}{2\pi} \times 12 = 4.39267429$



$\frac{1}{2}ab\sin h$

$\frac{35 - 27.6}{2} = 3.7$



$\frac{1}{2} \times 15 \times 7.5 \sin(0.4)$

5/10 marks

Part (a)

M1: Uses $l = r\theta$ with $l = 27.6$ and $\theta = 2.3$ correctly substituted in (line 2)

A1*: Rearranges the equation to achieve an expression for OC before proceeding to $OC = 12$ (condone lack of units)

Part (b)

M1: Attempts to subtract 2.3 from π. Seen on the right-hand side of their solution.

A1*: Achieves 0.421 (rad) with no errors seen, following a correct expression for angle AOB

Part (c)

M0: Does not use $A = \frac{1}{2}r^2\theta$ with $r = 12$ and $\theta = 2.3$

A0: Follows M0

B1: A correct expression or value for the length OB or OF which may be part of a calculation. 15.7 seen on the diagram of one of their triangles.

M0: Incorrect attempt to find the area of at least one of the two congruent triangles. They write $\frac{1}{2}\times 15\times 7.5\sin(0.4)$. The angle being rounded from 0.421 is condoned but they must use 15.7 and not 15.

dM0A0: Follows M0

Question 8 - Response C

(c) Find the total area of the stage, giving your answer to the nearest square metre.

(6)

oc = radius

$\theta r = 27.6 \qquad 2.3r = 27.6$

$r = 12$

$OC = 12m$

b) $AOG = \pi$

$AOB + GOF = \pi - 2.3 = \frac{0.8415}{2} = 0.421$

c) $35 - CDE =$

$35 - 27.6 = 7.4$

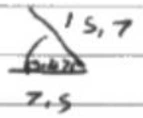
$BC + EF = 7.4 \qquad BC = 3.7$

$EF = 3.7$

$\vec{AO} = 7.5$

$OCDE = \frac{1}{2}\theta r^2 = \frac{1}{2}(2.3)(12^2) = 165.6$

$AOB = \frac{1}{2}ab\sin C$



$\frac{1}{2} \times 7.5 \times 15.7 \times \sin 0.421 = 24.06\cancel{1}$

$AOB = OGF \qquad OGF = 24.061$

$AOB + OGF + CDEO$

$24.061 + 24.061 + 165.6 = 213.7$

$= 214 m^2$

8/10 marks

Q8

i ? A B C

Part (a)

M1: Uses $l = r\theta$ with $l = 27.6$ and $\theta = 2.3$ correctly substituted in.

A0*: Does not rearrange the equation to achieve an expression for OC

Part (b)

M1: Attempts to subtract 2.3 from π

A0*: Poorly written jointed statement so A0*

Part (c)

M1: Attempts to use $A = \frac{1}{2}r^2\theta$ with $r = 12$ and $\theta = 2.3$. The values embedded in the formula is sufficient for this mark.

A1: 165.6 (or condone 166 if it has been rounded)

B1: A correct expression or value for the length OB or OF which may be part of a calculation (also seen on their diagram).

M1: Attempts to find the area of at least one of the two congruent triangles.

dM1: Solves the problem by adding the areas together to find the total area. It is dependent on the previous method marks and the B mark.

A1: awrt 214 (lack of units condoned). Must follow from a correct method.

Examiner comments

Part (a) and part (b) demonstrate how marks can be carelessly lost for either insufficient steps in their working, or for not writing separate mathematical statements.

Question 9

Question 9 - Introduction

This question tested knowledge of geometric sequences. In part (a), candidates were asked to establish the given result by setting up a valid equation using the first three terms of the sequence, which were in terms of *k*. In part (b), candidates were tested on their understanding of when a geometric sequence converges and to apply that knowledge to find the value in this case.

Question 9 - Question

9. The first three terms of a geometric sequence are

$$3k + 4 \qquad 12 - 3k \qquad k + 16$$

where k is a constant.

(*a*) Show that k satisfies the equation

$$3k^2 - 62k + 40 = 0$$

(2)

Given that the sequence converges,

(*b*) (i) find the value of k, giving a reason for your answer,

(ii) find the value of S_∞

(5)

(Total for Question 9 is 7 marks)

Question 9 - Mark Scheme

Question	Scheme	Marks	AOs
9(a)	$\dfrac{12-3k}{3k+4}=\dfrac{k+16}{12-3k}$	M1	3.1a
	$3k^2-62k+40=0$ *	A1*	1.1b
		(2)	
(b)(i)	$3k^2-62k+40=0 \Rightarrow k=...$	M1	1.1b
	States $k=20$ and gives a reason e.g. that this gives a values of r such that $\|r\|<1$	A1	3.2a
(ii)	$a=64$ **and** $r=-\dfrac{3}{4}$ (or allow $a=6$ **and** $r=\dfrac{5}{3}$)	B1	1.1b
	$S_\infty=\dfrac{"64"}{1-"\left(-\frac{3}{4}\right)"}=...$	M1	3.1a
	$S_\infty=\dfrac{256}{7}$	A1	1.1b
		(5)	
			(7 marks)

Question 9 - Examiner Comments

This was a more demanding than expected question on geometric sequences with a number of candidates getting confused with the value of k and the subsequent values of a and r required for the question. There was still access to later parts as the quadratic in k was given such that part (b) and part (c) could be attempted without part (a).

Most candidates were able to score full marks in part (a) by correctly giving an equation in k, almost always $\dfrac{12-3k}{3k+4}=\dfrac{k+16}{12-3k}$ and proceeding to the given quadratic with no errors seen. A noticeable number of candidates failed to gain the final mark as they gave a -40 term in their quadratic. Given that this was a printed answer, it was disappointing that candidates did not identify their error and correct it. Some candidates wrote down a correct expression for r but failed to create an equation, whilst a few candidates formed an equation for the sum of three terms, but invariably failed to expand and simplify correctly. There were several valid but more complicated efforts such as expressing the third term as the first term multiplied by the square of the ratio. This was more likely to result in errors and there were a significant number who did not know where to start. The common mistake where they could not set up an equation in r was to look for a common difference as if it was an arithmetic sequence.

Q9

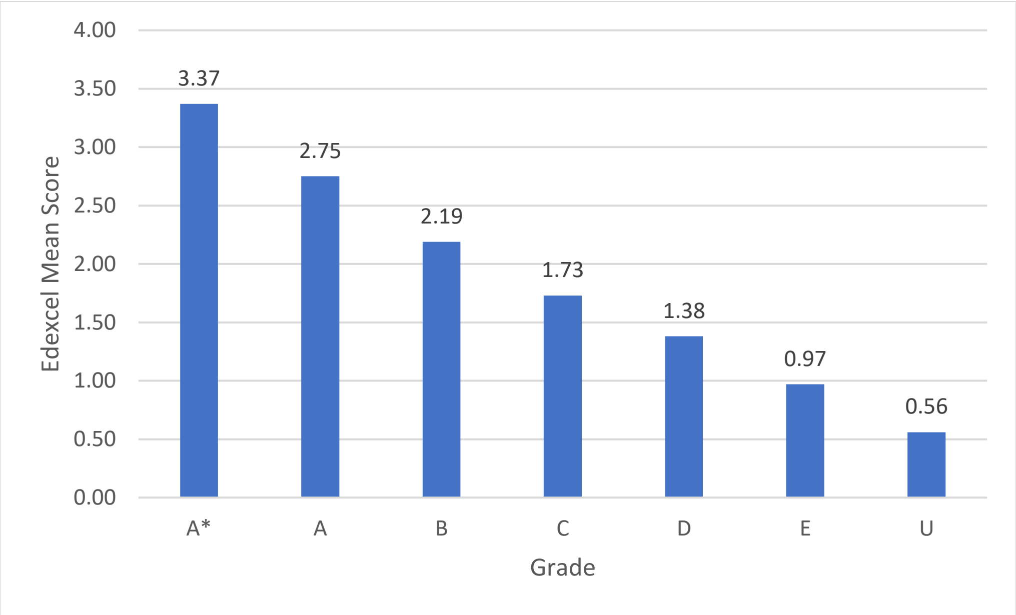
A
B
C

In part (b)(i) candidates were told that the given sequence converges. Candidates were required to find the value of *k* and to give a reason for their answer. Solving from calculator, or without working, was allowed and the majority of candidates were able to correctly solve the quadratic reaching the terms $k = 20$ and $k = \frac{2}{3}$. However, many found giving a reason particularly challenging. Of those who did score both marks, listing values in the sequence was a common explanation, as was finding the value of *r* and using the fact that $|r| < 1$. When attempting to reason via a calculation of the common ratio, there were often errors. A common response was to assume *k* was equal to *r* and then selecting $k = \frac{2}{3}$ as the final value. Some did not select a final value, keeping 20 and $\frac{2}{3}$. Some candidates who had chosen $k = 20$ gave comments such as "because it is an integer/whole number" or "because it is a constant". A significant number chose $k = 20$ giving a reason "because the sequence is convergent" which they had already been told and did not demonstrate an understanding of what the term means.

Part (b)(ii) proved to be very challenging for many candidates. It was quite often the case that candidates tried to use $r = \frac{2}{3}$ and just found the value $a = 6$ when $k = \frac{2}{3}$. It was very frequent that those who had correctly identified $k = 20$ in part (b)(i) did not show their working to find *a* and *r* and lost all marks in (b)(ii) as the sign error in substituting in $-\frac{3}{4}$ meant that it was unclear whether the correct value for *r* had been found, or if the correct sum to infinity formula was being used. Some candidates having found $\frac{256}{7}$ correctly then proceeded to give a rounded decimal as their answer which was condoned. Those who found $a = 6$ and $r = \frac{5}{3}$ were still able to score the B mark for a correct pair of values for *a* and *r*, even though they could not score the mark for using the sum to infinity formula with an invalid value for *r*.

Question 9 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	Max score	Mean %	A*	A	B	C	D	E	U
3.79	7	54%	6.21	5.04	3.91	2.84	1.89	1.11	0.54



Question 9 - Response A

a) $\dfrac{12-3k}{3k+4} = \dfrac{k+16}{12-3k}$

or $(12-3k)^2 = (k+16)(3k+4)$

or $144 - 72k + 9k^2 = 3k^2 + 52k + 64$

or $6k^2 - 124k + 80 = 0$

or $3k^2 - 62k + 40 = 0$

bi) $k = \dfrac{-(-62) \pm \sqrt{(-62)^2 - 4(3)(40)}}{2(3)}$

$k = 20$ or $k = \dfrac{2}{3}$

for sequence to converge, $|k| < 1$

$\therefore k = \dfrac{2}{3}$

bii) $a = 3\left(\dfrac{2}{3}\right) + 4$ or $a = 6$

$\dfrac{6}{1-\frac{2}{3}} = 18$

3/7 marks

Part (a)

M1: Forms a correct equation linking the three terms.

A1*: Achieves the given quadratic with at least one intermediate stage of working and no errors seen.

Part(b)

(i)

M1: Solves the given quadratic achieving at least one value for k.

A0: Selects $k = \frac{2}{3}$

(ii)

B0: Does not use $a = 64$ **and** $r = -\frac{3}{4}$ (**or** $a = 6$ **and** $r = \frac{5}{3}$)

M0: They use their value for k and not a valid value for r

A0: Follows M0.

Examiner comments

Many candidates confused $k = \frac{2}{3}$ with the value for r

Question 9 - Response B

$u_1 = 3k+4 \qquad u_2 = 12-3k \qquad u_3 = k+16$

$$\frac{12-3k}{3k+4} = \frac{k+16}{12-3k}$$

$$(12-3k)(12-3k) = (k+16)(3k+4)$$

$$144 - 36k - 36k + 9k^2 = 3k^2 + 4k + 48k + 64$$

$$\cancel{6k^2}\ 144 - 72k + 9k^2 = 3k^2 + 52k + 64$$

$$6k^2 - 124k + 80 = 0$$

$$\boxed{3k^2 - 62k + 40} \quad \div 2$$

k satisfies the equation

b) $3k^2 - 62k + 40$

$$\frac{\cancel{-}b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{62 \pm \sqrt{62^2 - 4(3)(40)}}{2(3)}$$

$k = 20 \quad , \quad k = \frac{2}{3}$

$3\left(\frac{2}{3}\right) + 4 = 6$

$12 - 3\left(\frac{2}{3}\right) = 10$

$\frac{2}{3} + 16 = \frac{50}{3}$

$3(20)+4=64$

$12-3(20)=-48$

$20+16=36$

$k=20$

~~$k=\frac{2}{3}$~~ because sequence converges

$k=\frac{2}{3}$ shows sequence diverging

$S_\infty = \frac{a}{1-r}$ $\quad a=6$

$r=\frac{5}{3}$

$\frac{6}{1-\frac{5}{3}} = -9$

Q9

A
B
C

4/7 marks

Part (a)

M1: Forms a correct equation linking the three terms.

A0*: Their final answer is missing the $=0$

Part (b)

(i)

M1: Solves the given quadratic achieving at least one value for k

A1: Chooses 20 and calculates at least two consecutive terms when k is 20

(ii)

B1: $a=6$ **and** $r=\frac{5}{3}$

M0: This mark can only be awarded if $|r|<1$

A0: Follows M0.

Examiner comments

Candidates should always check that they have achieved the given answer to make sure that they do not lose marks unnecessarily.

Question 9 - Response C

(5)

ar^{n-1}

a) $\frac{k+16}{12-3k} = \frac{12-3k}{3k+4}$

$(k+16)(3k+4) = (12-3k)(12-3k)$

$3k^2+4k+48k+64 = 144-36k-36k+9k^2$

$3k^2+52k+64 = 144-72k+9k^2$

$= 3k^2-42k+40=0$

$\boxed{k=20}, \; k=2/3$

$20 = 64, -48, 36$ ($-3/4$, $-3/4$)

$2/3 = 6, 10, 50/3$

$k=20$ because the 'r' value will be less than 1.

ii) $\frac{a}{1-r}$ $\quad \frac{64}{1--3/4} = 36.57$

6/7 marks

Part (a)

M1: Forms a correct equation linking the three terms.

A1*: Achieves the given quadratic with no errors including invisible brackets.

Part (b)

(i)

M1: Solves the given quadratic achieving at least one value for *k*.

A1: Chooses 20 and calculates at least two consecutive terms when *k* is 20. (They also give an argument about *r* being less than 1 which would have scored the mark.)

(ii)

B1: $a = 64$ **and** $r = -\frac{3}{4}$

M1: A full attempt to find S_∞ by using their value of k to reach a value for r such that $|r| < 1$ and a value for a

A0: Incorrect

Examiner comments

The candidate lost the mark in (b)(ii) because 36.57 is not the value of the sum to infinity.

Question 10

Introduction | Question | Mark Scheme | Examiner Comments
Performance | Response A | Response B | Response C

Question 10 - Introduction

This question tested knowledge of the equation of the circle in part (a), in order to find the coordinates of the centre of C and the radius of C, often by completing the square. In part (b), candidates were required to find the range of possible values of k such that a given line would intersect C at 2 distinct points. This tested understanding of the discriminant and being able to solve a quadratic inequality.

Question 10 - Question

10. A circle C has equation

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$

where k is a constant.

(a) Find in terms of k,

(i) the coordinates of the centre of C

(ii) the radius of C

(3)

The line with equation $y = 2x - 1$ intersects C at 2 distinct points.

(b) Find the range of possible values of k.

(6)

(Total for Question 10 is 9 marks)

Question 10 - Mark Scheme

Question	Scheme	Marks	AOs
10(a)(i)	Centre $(-3k,\ k)$	B1	2.2a
(ii)	$(x+3k)^2-9k^2+(y-k)^2-k^2+7=0 \Rightarrow (x+3k)^2+(y-k)^2=...$	M1	1.1b
	Radius $\sqrt{10k^2-7}$	A1ft	2.2a
		(3)	
(b)	$x^2+(2x-1)^2+6kx-2k(2x-1)+7=0 \Rightarrow ...x^2+(pk+q)x+rk+s(=0)$	M1	1.1a
	$5x^2+(2k-4)x+2k+8\ (=0)$	A1	1.1b
	$(2k-4)^2-4\times5\times(2k+8)=0 \Rightarrow k=...$	dM1	2.1
	Critical values $= 7\pm\sqrt{85}$	A1	1.1b
	$k<"7-\sqrt{85}"$ or $k>"7+\sqrt{85}"$ o.e.	ddM1	3.1a
	$k<7-\sqrt{85}$ or $k>7+\sqrt{85}$ o.e.	A1	2.5
		(6)	
			(9 marks)

Question 10 - Examiner Comments

Candidates were typically able to make good progress in part (a) with the method of completing the square being familiar to many when dealing with the equation of the circle. Part (b) was focused on using the discriminant and solving a quadratic inequality, however, the manipulation proved to be a lot for some, resulting in a wide range of marks. Again, the access was good on this question such that candidates who could not do part (a) were not restricted from attempting part (b), although those with errors from part (a) often found that this caused issues in part (b) and resulted in losing many of the marks.

In general candidates obtained $(-3k,\ k)$ as the centre of the circle in part (a). Common errors included $(3k,\ -k)$ and $(-3kx,\ ky)$. If candidates were successful at completing the square, they could find the correct radius of the circle. When attempting to complete the square a common error was to subtract $3k^2$ instead of $(3k)^2$. Occasionally, candidates would find $r^2=10k^2-7$ but did not proceed to take the square root. Those who collected terms on the right-hand side of the equation in order to form the square of the radius made fewer sign errors than those who did not. Others tried to manipulate the equation mentally, so writing down a wrong radius with no evidence of its development (and thus no method mark). One error was to write $\sqrt{10k^2-7}=\sqrt{10}k-\sqrt{7}$. Sometimes there were sign slips such as writing the -7 as +7 when rearranging the equation.

Q10
i
?
A
B
C

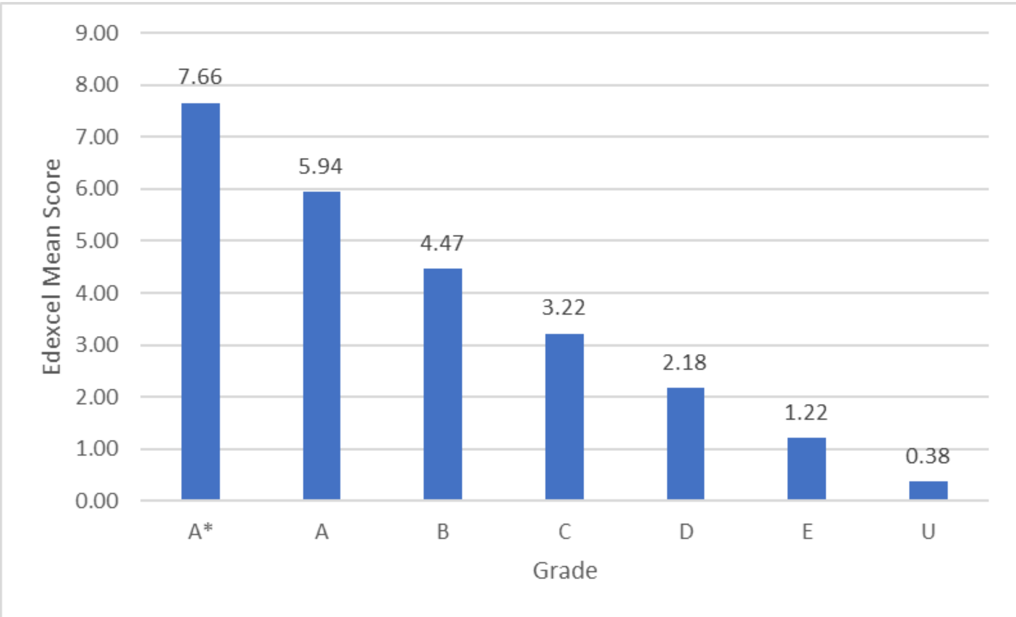
In part (b), whilst a number of candidates had little idea how to proceed, the vast majority did realise what was required and attempted to produce an equation in x by substitution. Many lost their way in the algebraic complexity, or did not know what to do with the equation (sometimes trying to solve it to find a value for k). A significant number of candidates did produce a three-term quadratic for x, with coefficients in terms of k.

Centres should note that the scheme required the quadratic in x to be of the form specified, and that it be set out in a way which identified the coefficients (possibly implied by later use in the discriminant). So, for example, "$(2k-4)x$" was required rather than "$2kx-4x$" as this provided evidence of recognising that the coefficients were the key to solving this problem. Quite a few candidates had a constant term which was not of the required form which meant that no marks could be scored.

In general candidates who found critical values for their discriminant by solving $b^2-4ac=0$ were more successful than those who expressed it as an inequality. Often the inequality was simply carried forward to a statement about k; those who identified the two critical values by solving an equation perhaps thought more carefully about where the region was. The "need" to write a single inequality was seemingly pressing for many, so $7+\sqrt{85}<k<7-\sqrt{85}$ was seen. A few candidates did not use exact values of k and lost the final accuracy mark. Some candidates used 'and' with their two regions, however, most recognised that this needed to be 'or', either stating this explicitly or just using a comma between their two answers. A number of capable candidates brought some interesting approaches to bear on the problem, adopting a variety of techniques from Further Maths specifications.

Question 10 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	Max score	Mean %	A*	A	B	C	D	E	U
4.47	9	50%	7.66	5.94	4.47	3.22	2.18	1.22	0.38



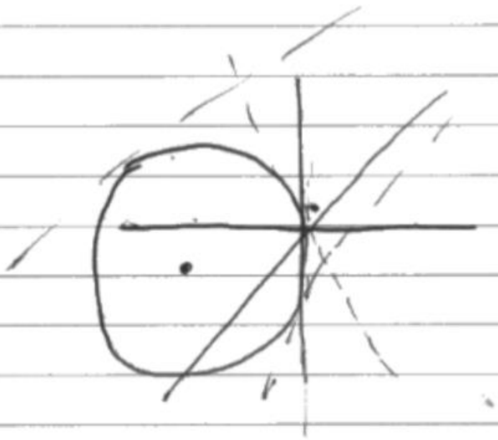
Q10
A
B
C

Question 10 - Response A

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$
$$x^2 + 6kx + y^2 - 2ky + 7 = 0$$
$$(x+3k)^2 - 9k^2 + (y-k)^2 - k^2 + 7 = 0$$
$$(x+3k)^2 + (y-k)^2 = 10k^2 - 7$$

Centre $= (-3k, k)$

radius $= \sqrt{10k^2 - 7}$



$y = 2x - 1$

$y + 1 = 2x$

$x = \frac{y+1}{2}$

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$
$$x^2 + y^2 + 7 = -6kx + 2ky$$
$$x^2 + y^2 + 7 = k(-6x + 2y)$$
$$k = \frac{x^2 + y^2 + 7}{-6x + 2y}$$

2 distinct Points

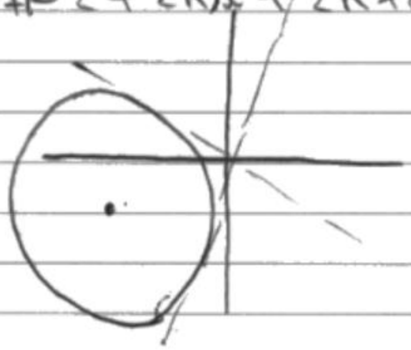
$b^2 - 4ac > 0$

$x^2 + (2x-1)^2 + 6xk - 2k(2x-1) + 7 = 0$

$x^2 + 4x^2 - 2x + 1 + 6xk - 4xk + 2k + 7 = 0$

$\underbrace{5x^2}_{a} \underbrace{- 2x + 2xk}_{b} \underbrace{+ 2k + 8}_{c}$

$5x^2 + (-2+2k)x + 2k + 8 = 0$



$\cancel{b^2 +}$

$b^2 - 4ac = 0$

$\cancel{(-2x+2k}$

$(-2+2k)^2 - 4(5)(2k+8) = 0$

$4k^2 - 8k + 4 - 40k + 160 = 0$

$4k^2 - 48k + 164 = 0$

4/9 marks

Part (a)(i)

B1: $(-3k, k)$

Part (a)(ii)

M1: Attempts to find r^2 (seen on the right-hand side of their equation for the circle)

A1ft: $\sqrt{10k^2 - 7}$

Part (b)

M1: Substitutes $y = 2x - 1$ into the equation of the circle and attempts to collect terms proceeding to a three-term quadratic in x of the required form.

A0: Incorrect

dM0: Attempts to find $b^2 - 4ac$ for their three-term quadratic but not does not attempt to find at least one critical value.

A0ddM0A0: Follows

Question 10 - Response B

a) $(x+3k)^2 - 9k^2 + (y-ky)^2 - k^2 + 7 = 0$

i) $(-3k, -k)$

ii) $10k^2 - 7 = r^2$, $r = \sqrt{10k^2-7}$

b) $x^2 + (2x-1)^2 + 6kx - 2k(2x-1) + 7 = 0$

$5x^2 - 4x + 1 + 6kx - 4kx - 2k + 7 = 0$

$5x^2 - 4x + 2kx - 2k + 8 = 0$

$b^2 - 4ac = 0$

$(-4+2k)^2 - 4(5)(-2k+8) = 0$

$4k^2 - 16k + 16 - 20(-2k+8)$

$4k^2 - 16k + 16 + 40k - 160 = 0$, $4k^2 + 24k - 144$

$-3-3\sqrt{5} > k > -3 \pm 3\sqrt{5}$

5/9 marks

Part (a)(i)

B0: Incorrect

Part (a)(ii)

M1: Attempts to find r^2 (seen at the start of (ii))

A1ft: Correct (condone the slightly shorter square root notation)

Part (b)

M1: Substitutes $y = 2x - 1$ into the equation of the circle and attempts to collect terms proceeding to a three-term quadratic in x of the required form (implied by their values for a, b and c within the discriminant)

A0: Incorrect

dM1: Attempts to find $b^2 - 4ac$ for their three-term quadratic and attempts to find at least one critical value.

A0: Incorrect

ddM1: Attempts to find the outside region for their values (condone this expression for this mark)

A0: Follows A0

Question 10 - Response C

10a) $(x+3k)^2 - 9k^2 + (y-k)^2 - k^2 + 7 = 0$

$(x+3k)^2 + (y-k)^2 = 10k^2 - 7$

a i) $(-3k, k)$

ii) $\sqrt{10k^2 - 7}$

b) $x^2 + (2x-1)^2 + 6kx - \cancel{\quad} + 2k(2x-1) + 7 = 0$

$\cancel{x^2 + 4x^2 - 4x + 1 + 6kx} \; \cancel{-4kx} + 2k + 7 = 0$

$5x^2 - 4x + 2kx + (8 + 2k) = 0$

$5x^2 + (-4 + 2k)x + (8 + 2k) = 0$

$b^2 - 4ac > 0$

$(2k-4)^2 - 4(5)(8+2k) > 0$

$4k^2 - 16k + 16 - 20(8+2k) > 0$

$4k^2 - 16k + 16 - 160 - 40k > 0$

$\cancel{4k^2 - 16k}$

$4k^2 - 56k - 144 > 0$

$k = 7 + \sqrt{85}$ or $k = 7 - \sqrt{85}$

$7 - \sqrt{85} < k < 7 + \sqrt{85}$

7/9 marks

Q10
i
?
A
B
C

Part (a)(i)

B1: $(-3k,\ k)$

Part (a)(ii)

M1: Attempts to find r^2 (seen on the right-hand side of their equation for the circle)

A1ft: $\sqrt{10k^2-7}$

Part (b)

M1: Substitutes $y = 2x - 1$ into the equation of the circle and attempts to collect terms. proceeding to a three-term quadratic in x of the required form.

A1 $5x^2 + (-4+2k)x + 2k + 8 \ (=0)$

dM1: Attempts to find $b^2 - 4ac$ for their three-term quadratic and attempts to find at least one critical value.

A1: $7 \pm \sqrt{85}$

ddM0: Attempts to find the inside region.

A0: Follows ddM0

Question 11

Question 11 - Introduction

This modelling question tested understanding of exponential growth and the associated linear relationship when using a logarithmic graph. Candidates were then asked in part (c) to use the given information to evaluate the reliability of the model.

Question 11 - Question

11.

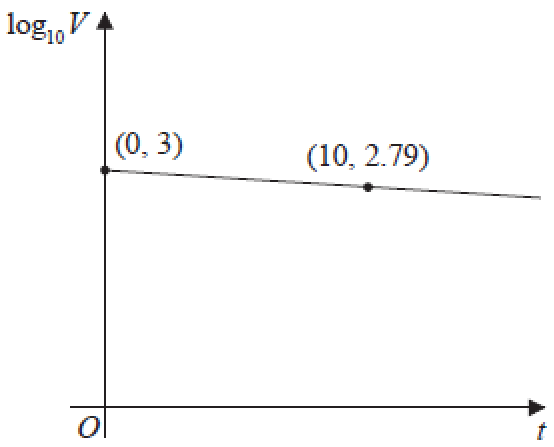


Figure 2

The value, V pounds, of a mobile phone, t months after it was bought, is modelled by

$$V = ab^t$$

where a and b are constants.

Figure 2 shows the linear relationship between $\log_{10} V$ and t.

The line passes through the points (0, 3) and (10, 2.79) as shown.

Using these points,

(*a*) find the initial value of the phone,

(2)

(*b*) find a complete equation for V in terms of t, giving the exact value of a and giving the value of b to 3 significant figures.

(3)

Exactly 2 years after it was bought, the value of the phone was £320

(*c*) Use this information to evaluate the reliability of the model.

(2)

(Total for Question 11 is 7 marks)

Question 11 - Mark Scheme

Question	Scheme	Marks	AOs
11(a)	$\log_{10} V = 3 \Rightarrow V = 10^3$	M1	1.1b
	$(V =)$ £1000	A1	3.4
		(2)	
(b)	e.g. $(\log_{10} b =) \dfrac{2.79-3}{10-0} = -0.021$ **or** $\log_{10} V = 3 - 0.021t$ **or** $10^{2.79} = \text{"}1000\text{"}b^{10}$	M1	1.1b
	e.g. $b = 10^{-0.021}$ $(= 0.952796....)$ **or** $V = 10^3 \times 10^{-0.021t}$ **or** $b = \sqrt[10]{\text{"}0.61659...\text{"}}$	M1	3.1b
	$V = 1000 \times 0.953^t$	A1ft	3.3
		(3)	
(c)	e.g. $V = 1000 \times \text{"}0.953\text{"}^{24}$ $(= £315)$ or e.g. $\log_{10} V = 3 - \text{"}0.021\text{"} \times 24 \Rightarrow V = ...(= £313)$	M1	3.4
	which is close (to £320) so it is a suitable model	A1	3.2b
		(2)	
			(7 marks)

Question 11 - Examiner Comments

This question proved to be quite challenging for a number of candidates, however those who were able to manipulate logarithms using the rules correctly, along with calculating the gradient, were often very successful. It is worth noting that there were a number of candidates who did not pay attention to the labelling on the *y*-axis and therefore were unable to score any marks on this question. As with many modelling questions, candidates currently lack proficiency at relating the mathematics to the context of the question.

In part (a), most candidates were able to find the initial value correctly, although several lost the accuracy mark by forgetting to include units. The location of the unit was condoned e.g. 1000£ was seen on many occasions. A common reason for candidates not achieving any marks was to assume $V = 3$ or $V = \log_{10} 3$, instead of $\log_{10} V = 3$. A few candidates used natural logs instead of $\log_{10}$, resulting in $V = e^3$ instead of $V = 10^3$. Some candidates just stated the initial value was 3, not fully appreciating what information the linear graph was representing.

Q11
A
B
C

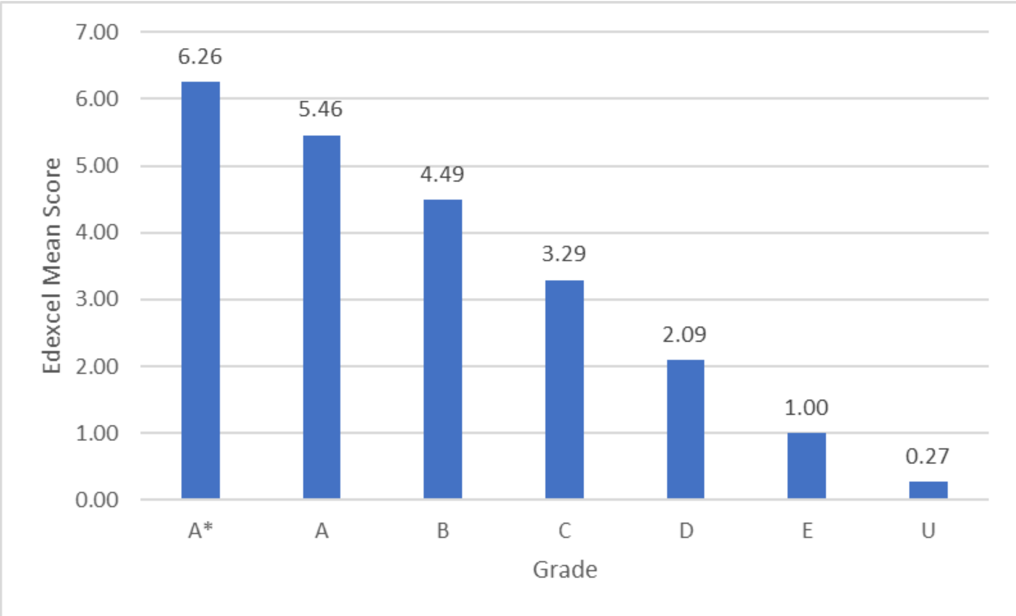
Part (b) was discriminating, requiring some understanding of the relationship between $V = ab^t$ and the linear graph. Those candidates who were most successful with this part of the question, made the link between part (a) and the value of *a* without having to do additional calculations. Many candidates were able to find the gradient of the graph correctly and pick up the first method mark. However, finding a value for *b* was difficult for many candidates. Those candidates who did not make the link with part (a) often had to recalculate *a* or attempted to use simultaneous equations, often with little success. There were a lot of very convoluted incorrect solutions in this question which showed a poor grasp of logarithms and the associated rules; $t\log ab$ was commonly seen; others failed to correctly relate $\log V = \log a + t\log b$ to $y = mx + c$ despite previous questions on the topic, confusing which were the variables and which were the constants. There were several cases of candidates mixing up the *x*, *y* coordinates in their substitution. It was encouraging to see that most candidates tried to give the full equation if they found values for *a* and *b*, but there was still a good number that did not and thus, lost the final mark. A few candidates were unable to score the final mark for leaving their answer as $\log_{10} V = 3 - 0.021t$. Candidates were able to gain the final mark of part (b), if the complete equation was seen in part (c), although this was rare to see.

In part (c), candidates who used $t = 24$ generally scored the method mark even if their model was incorrect but of the correct form. The most common form used was a continuation of their answer to part (b), usually $V = ab^t$ where *a* was positive. The majority of candidates were able to make a valid comparison accompanied by a sufficient explanation to earn the final accuracy mark. Some calculated the percentage error to justify their thinking which was acceptable. A minority thought the small difference was too great for the model to be reliable.

A few candidates used the method of substituting $t = 24$ into an equation of the form $\log_{10} V = p + qt$ to find a value for $\log_{10} V$. The discerning candidates either compared their $\log_{10} V$ with $\log_{10} 320$, or they proceeded to find *V* and compared this with 320. This method was slightly more complicated, and the success rate of this method was mixed. The units in this question caught a considerable number of candidates out, when checking the suitability of the model and substituting in 2 (years) rather than 24 (months), highlighting the importance of reading the question carefully. This led to a conclusion that the model was unreliable due to the vastly different amount that was yielded. Candidates did not consider the fact that this could mean that there was an error in their own calculation.

Question 11 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	Max score	Mean %	A*	A	B	C	D	E	U
4.09	7	58%	6.26	5.46	4.49	3.29	2.09	1.00	0.27



Question 11 - Response A

a) $= t = 0$

$3 = ab^0$

$\log_{10} = 3$

10^3

a) 1000

b) $\frac{2.79 - 3}{10 - 0} = -0.021$

$b = -0.021$

$V = 1000 \times (-0.021)^t$

c) $1000 \times (-0.021)^2$

$= 0.441$

↳ making the model not very reliable

2/7 marks

Part (a)
M1: 10^3
A0: 1000 (missing units)
Part (b): Mark (b) and (c) together
M1: Attempts to find the gradient between the two points.
M0: Incorrect method to find *b*
A0ft: Follows M0.

Q11 A B C

Part (c)

M0: No attempt to substitute $t = 24$ or $V = 320$ into an equation.

A0: Follows M0.

Examiner comments

A significant number of candidates did not appreciate the model was in months whereas part (c) referred to 2 years (requiring the use of 24 months)

Question 11 - Response B

(4)

a) $V = ab^t$

$\log V = \log a + t \log b$

$3 = \log a + (0)\log b$

$a = 10^3 = 1000$ pounds

$\cancel{3 = ab^0}$ $\quad 10^3 = ab^0$

$\cancel{a = 3}$ $\quad a = 10^3 = 1000$ pounds

initial value of phone is 1000 pounds

b) $V = ab^t$

$\log_{10} V = \log_{10} ab^t$

$\log_{10} V = \log_{10} a + \log_{10} b^t$

$\log_{10} V = \log_{10} a + t\log_{10} b$

$3 = \log_{10} a$

$a = 10^3 = 1000$

$\cancel{10 = 1000 + 2.79 \log_{10} b}$

$\cancel{2.79 = 1000 + 10\log_{10} b}$

$\cancel{-997.21 = 10\log_{10} b}$

$\cancel{-99.721 = \log_{10} b}$

$\cancel{b = 10^{-99.721} = 0}$

$\cancel{2.79 = \log_{10} 10^3}$

$2.79 = \log_{10} 10^3 + 10 \log_{10} b$

$2.79 = 3 + 10\log_{10} b$

$-0.21 = 10 \log_{10} b$

$\frac{-0.21}{10} = \log_{10} b$

$b = 10^{-\frac{0.21}{10}} = 0.953$ (3 s.f.)

$\log_{10} V = \log_{10} 1000 + t\log_{10} 0.953$

c) $\log_{10} V = 3 + 2\log_{10} 0.953 = 2.9581\ldots$

$V = 10^{2.958\ldots} = \cancel{90.7}$ £907.80 to 2d.p.

the model is not reliable as £907.80 is way bigger than £320.

Q11 i ? A B C

4/7 marks

Part (a)

M1: 10^3 seen

A1: 1000 pounds

Part (b): Mark (b) and (c) together

M1: Scored for the equation $2.79 = \log_{10} 10^3 + 10\log b$

M1: Attempts to find a value for b using their equation.

A0ft: Equation not in required form.

Part (c)

M0: Substitutes $t = 2$ rather than $t = 24$ into their model.

A0: Follows M0.

Examiner comments

Part (b) required a complete equation for V in terms of t. Candidates should be encouraged to check that they have answered the question and given their answer in the required form.

Q11

A

B

C

Question 11 - Response C

a. $\log_{10} V = 3$

$V = £1000$

b. $V = ab^t$

$\log V = \log ab^t$

$\log V = \log a + t\log b$

$\log a = 3$

$a = 1000$

$\log V = \log 1000 + t\log b$, when $t = 10$

$2.79 = \log 1000 + 10\log b$

$10\log b = -0.29$ $\log b = -0.021$ $b = 0.953$

$\log_{10} V = \log_{10} 1000 + \cancel{}\, t \log_{10} 0.953$

c. $\log_{10} V = \log_{10} 1000 + t \log_{10} 0.953$

After 24 months:

$\log_{10} V = \log_{10} 1000 + 24 \times \log_{10} 0.953$

$\log_{10} V = 3 - 0.502$

$\log_{10} V = 2.498$

$V = 314.9$

Model is quite ~~retia~~ reliable, $\left(1 - \frac{314.9}{320}\right) \times 100 = 1.594\%$

Only a 1.594% percentage error

6/7 marks

Part (a)

M1: Sets $\log_{10} V = 3$ and attempts to find a value for *a*, implied by the correct answer.

A1: £1000 (includes units)

Part (b): Mark (b) and (c) together

M1: Scored for the equation, which is equivalent to $2.79 = 3 + 10\log b$ (line 7 of (b))

M1: Attempts to find a value for *b* using their equation. The copying error is recovered by a correct equation to find *b* and then proceeds to the correct value for *b*.

A0ft: Equation not in required form.

Part (c)

M1: Substitutes $t = 24$ into their model of the form $\log_{10} V = p + qt$ where *p* is positive and finds a value for *V*.

A1: Reaches a value for *V* which is awrt £313-£315 and uses percentage error, which is correct for their values, to compare (to £320). Concludes that the model is reliable.

Q11

A
B
C

Question 12

Introduction | Question | Mark Scheme | Examiner Comments

Performance | Response A | Response B | Response C

Question 12 - Introduction

This question required candidates to demonstrate differentiation from first principles to establish that the derivative of sin *x* is cos *x*. Candidate were given the limiting values as *h* tends to zero and they were able to use the formula for $\sin(A \pm B)$ without proof.

Question 12 - Question

12. $$y = \sin x$$

where *x* is measured in radians.

Use differentiation from first principles to show that

$$\frac{dy}{dx} = \cos x$$

You may

- use without proof the formula for $\sin(A \pm B)$
- assume that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

(Total for Question 12 is 5 marks)

Question 12 - Mark Scheme

Question	Scheme	Marks	AOs
12	$\dfrac{\sin(x+h)-\sin x}{h}$	B1	2.1
	$\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$	M1	1.1b
		A1	1.1b
	(As $h \to 0$), $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right) \to 0\times\sin x+1\times\cos x$	dM1	2.1
	so $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\cos x$ *	A1*	2.5
			(5 marks)

Question 12 - Examiner Comments

Q12
i
?
A
B
C

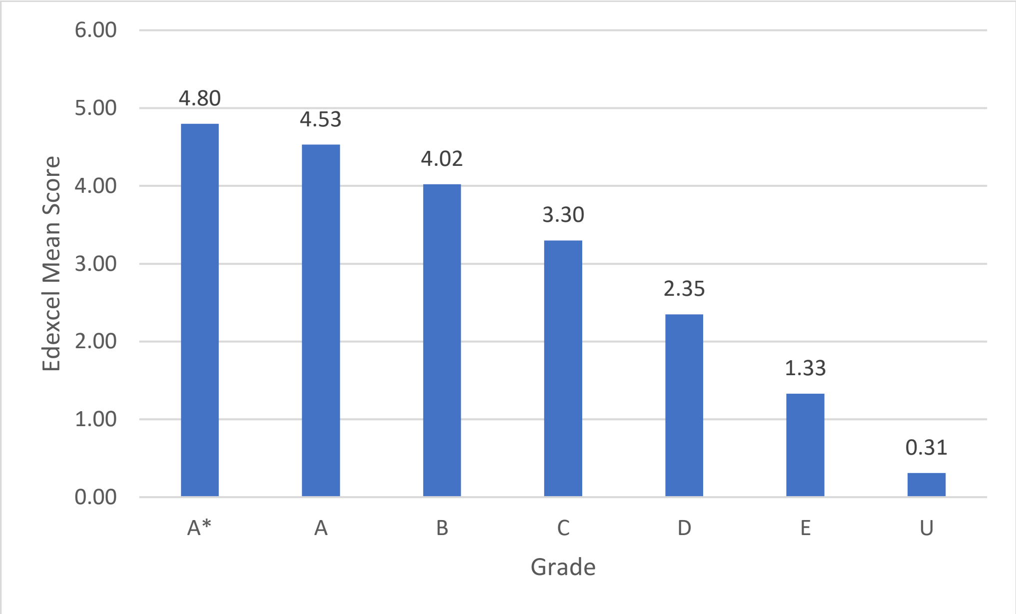
This differentiation from first principles question was similar to the one that appeared in 2018 and candidates demonstrated a much greater confidence with the way in which they should approach this topic. The limits were provided in the question and the accessibility was evident as it was very rare to see a completely blank script, which is not always true for a differentiating from first principles question. Most candidates were able to obtain the first 3 marks of this question, with very few sign errors seen when using the addition formula. Most correctly started with the fraction for the gradient of the chord $\dfrac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$ as given in the formula booklet. Occasionally all a candidate would attempt was the addition formula in terms of *A* and *B*, but more often than not it was in the correct format and correct.

Scoring the last two marks was not as common, but still achieved more often than not. There were a variety of layouts used for the candidates to justify the replacement of terms with 0 and 1. If candidates lost marks here, it was often because they failed to separate their terms in *h* correctly. Some candidates introduced extra *h*'s, forming expressions like $\cos xh\times\sin hh$. At other times, candidates simply tried to cancel terms incorrectly or stopped. Notation was generally good with limiting arguments correct. The main mistakes that resulted in lost marks were either failing to isolate the required expressions and jumping to the answer or trying to replace each of the 3 terms separately, failing to spot the common factor of sin *x* in two of them. There were also several candidates incorrectly splitting up the product of each term resulting in an extra *h* in the denominators. Incorrect notation resulted in the loss of the final mark, in particular the limit notation was often seen right through to the end, in the final answer, even after the limits had been used.

A small number of candidates proceeded by using the small angle formula which was generally done well; where errors were seen it was generally not cancelling correctly. One surprising thing to note was the number of candidates who felt it acceptable to not clearly identify the numerator of the fraction by using very small fraction lines, resulting in their solution looking like only one term was being divided by *h*. Many candidates failed to write $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\cos x$ at the end, despite this being what they had been asked to show. If $f'(x)$ had not been acceptable as an answer, then many candidates would have lost the final A mark. Only a very small handful of candidates used small angle approximations.

Question 12 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
3.60	5	72%	4.80	4.53	4.02	3.30	2.35	1.33	0.31



Question 12 - Response A

$$y = \sin x$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\sin x + h - \sin x}{h}.$$

$$\lim_{h \to 0} \frac{\sin x + h}{h} - 1$$

$$\lim_{h \to 0} \quad 1 + \frac{h}{h} - 1 \qquad 1 + 1 - 1 = 1$$

1/5 marks

B1: Gives the correct fraction $\dfrac{\sin(x+h) - \sin x}{h}$

M0: Does not use the compound angle for $\sin(x \pm h)$

A0: Follows M0.

dM0: Follows M0 as this is a dependent mark on both the B and the previous M mark.

A0: Follows M0.

Examiner comments

This candidate made a good start by quoting the equation from the formula booklet. However, a candidate who either did not start off with the correct fraction or did not use the compound angle formula would not be able to score the final three marks on this question.

Q12

A

B

C

Question 12 - Response B

$y = \sin x$

$\frac{dy}{dx} = \cos x$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} = \lim_{h \to 0} \frac{\sin x \cosh + \cos x \sin h - \sin x}{h} =$

$= \lim_{h \to 0} \frac{\sin x(\cosh - 1) + \cos x \sin h}{h} = \lim_{h \to 0} \frac{\sin h}{h} \to 1$

$\frac{\cosh - 1}{h} \to 0 \qquad \therefore f'(x) = \cos x$

3/5 marks

B1: Correct fraction $\frac{\sin(x+h) - \sin x}{h}$

M1: Uses the compound angle formula for $\sin(x \pm h)$ to give $\sin x \cos h \pm \cos x \sin h$

A1: $\frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$ or equivalent. For this mark the limit notation can either be present or not.

dM0: Incomplete attempt to apply the given limits to the gradient of their chord by isolating $\left(\frac{\cos h - 1}{h}\right)$ and $\left(\frac{\sin h}{h}\right)$ and replacing with 0 and 1 respectively. Proceeding from $\frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$ to the given answer is not sufficient to imply the method of applying the given limits.

A0*: Follows dM0

Question 12 - Response C

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$\lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$\lim_{h \to 0} \frac{\sin h(\cos x)}{h} + \frac{\sin x(\cos h - 1)}{h}$$

$$\lim_{h \to 0} \cos x + 0(\sin x)$$

$$\therefore \frac{dy}{dx} = \cos x$$

5/5 marks

B1: Correct fraction $\frac{\sin(x+h)-\sin x}{h}$

M1: Uses the compound angle formula for $\sin(x \pm h)$ to give $\sin x \cos h \pm \cos x \sin h$

A1: $\frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$ or equivalent. For this mark the limit notation can either be present or not.

dM1: Complete attempt to apply the given limits to the gradient of their chord by isolating $\left(\frac{\cos h - 1}{h}\right)$ and $\left(\frac{\sin h}{h}\right)$ and replacing with 0 and 1 respectively. As the terms have been isolated, the 1 can be implied for $1 \times \cos x$

A1*: Uses correct mathematical language of limiting arguments to show that $\frac{dy}{dx} = \cos x$ with no errors seen. (all previous marks must also have been scored)

Examiner comments

For the final mark to be awarded all previous marks needed to be scored, we needed to see $h \to 0$ at some point in their solution and linking $\frac{dy}{dx}$ with $\cos x$. A lot of candidates appeared to be unsure when to write e.g. $\lim_{h\to 0}$ and when they did not need to write it anymore. The lack of brackets around the expression the limit was referring to was also condoned. On this occasion, the position of where this type of notation was, was condoned, other than if it appeared in their final answer e.g. $\frac{dy}{dx} = \lim_{h\to 0}(\cos x)$ but candidates should aim to develop the mathematical fluency of their solutions, particularly for this type of question as less coherent solutions may not be condoned on similar questions in the future.

Q12

A

B

C

Question 13

Introduction | Question | Mark Scheme | Examiner Comments | Performance | Response A | Response B | Response C

Question 13 - Introduction

This modelling question tested a candidate's ability to be able to model a rollercoaster scenario using two different models (one quadratic and one trigonometric). Candidates were asked to find these complete models in parts (a) and (c), whilst in part (b) candidates were asked to determine the height at a particular point in time. In part (d) candidates were asked to give a reason why the alternative model would be more appropriate, given that the carriage moved continuously for 2 minutes.

Question 13 - Question

13. On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m
- a carriage starts a circuit at a vertical height of 2 m above the ground
- the ground is horizontal

The vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where a and b are positive constants.

(*a*) Find a complete equation for the model.

(3)

(*b*) Use the model to determine the height of the carriage above the ground when $t = 40$

(1)

In an alternative model, the vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is given by

$$H = 29\cos(9t + \alpha)° + \beta \qquad 0 \leqslant \alpha < 360°$$

where α and β are constants.

(*c*) Find a complete equation for the alternative model.

(2)

Given that the carriage moves continuously for 2 minutes,

(*d*) give a reason why the alternative model would be more appropriate.

(1)

(Total for Question 13 is 7 marks)

Question 13 - Mark Scheme

Question	Scheme	Marks	AOs
13(a)	$a = 60$	B1	3.1b
	$2 = \text{"}60\text{"} - b(-20)^2 \Rightarrow b = \ldots$	M1	3.4
	$H = 60 - 0.145(t-20)^2$	A1	3.3
		(3)	
(b)	Height = 2 m	B1	3.4
		(1)	
(c)	$\alpha = 180$ or $\beta = 31$	M1	3.4
	$H = 29\cos(9t+180)° + 31$	A1	3.3
		(2)	
(d)	e.g. "The model allows for more than one circuit"	B1	3.5a
		(1)	
			(7 marks)

Question 13 - Examiner Comments

A significant number of candidates found this question very challenging, with a fair proportion of blank answers, and another significant number working with various substitutions in (a) that were quickly abandoned. Those who were able to work with both the quadratic and trigonometric models confidently were able to score highly and demonstrated a good understanding of how mathematics can be used to model real life contextual problems.

In part (a), there were two common approaches that led to successfully scoring full marks. One was applying the knowledge that $b(t-20)^2 = 0$ when $H = 60$ which therefore meant $a = 60$. This then enabled the candidate to substitute $H = 2,\ t = 0$ and find a complete equation. It was clear when candidates were able to apply this knowledge, and knowing this gave them a huge advantage in this question. Noticing that b was positive was pivotal when taking this approach. The other approach that led to successful answers involved multiplying out the brackets, and differentiating with respect to t. This enabled the candidate to find that $t = 20$ when $\frac{\mathrm{d}H}{\mathrm{d}t} = 0$, thus overcoming a shortfall in knowledge over when geometrically the maximum occurs in a negative quadratic. This second approach led to success less often, and candidates often ended up with a page of various quadratic expressions and equations that looked costly in terms of time and did not yield marks.

An overwhelming majority of candidates who scored full marks in (a) successfully found H to be 2 m in part (b). Some candidates still managed to find this value with either an incorrect answer in (a) or no answer at all, by perhaps noticing that due to the symmetry of the quadratic, the height after 40 seconds was the same as at the start, which they are told was 2 m in the question.

In part (c), a significant number of candidates had no real idea how to start this. Many attempts involved fruitlessly expanding $\cos(9t+\alpha)$ but most candidates who successfully found α and β did not do this. The candidates who realised that when $H = 60$, $\cos(9t+\alpha) = 1$ which meant $\beta = 31$ had the most success. Candidates who did this typically then substituted $t = 20$ into $\cos(9t+\alpha) = 1$ to find a value of α. Candidates who found one value typically found the other. There were a number of candidates who found the two values that did not 'find a complete equation' as the question required. However, there were a pleasing number of fully correct solutions, and almost all candidates who found α and β continued to give the full model.

In part (d), the most popular correct answers involved an identification of the cyclic nature of the trigonometric function, or realising the first model would become very negative. A significant number of incorrect answers involved candidates saying the original model would keep increasing over time, which showed a lack of understanding of the negative quadratic model. Others made reference to the alternative model being continuous which was too vague. Some candidates managed to score this mark without scoring any other mark on this question which was, again, pleasing to see candidates attempting later parts of questions even if they have made little progress on earlier parts.

Q13

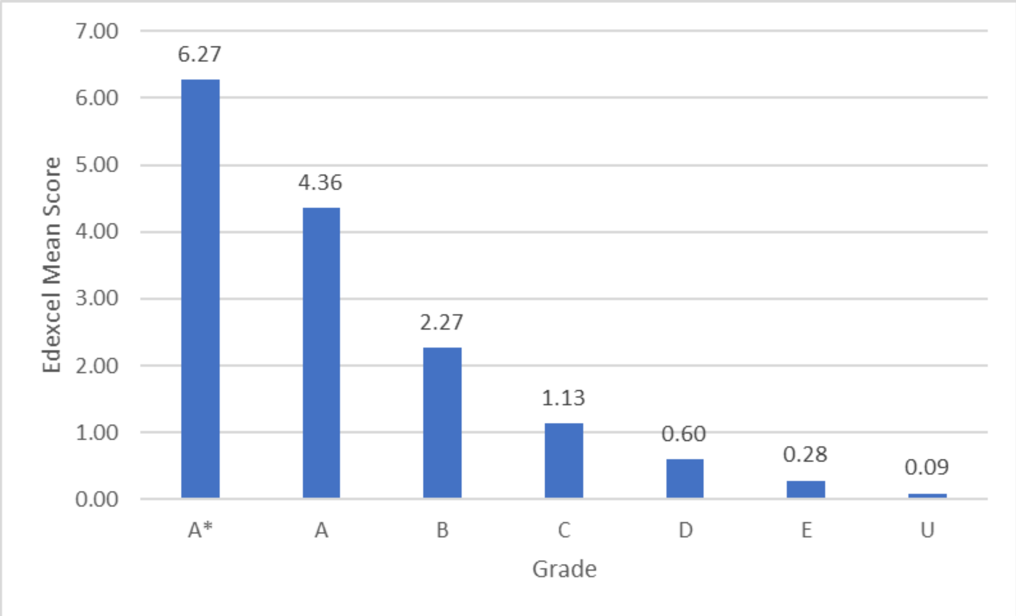
A

B

C

Question 13 - Performance

Mean score	Max score	Mean %	Edexcel averages: mean scored by candidates achieving grade: A*	A	B	C	D	E	U
2.80	7	40%	6.27	4.36	2.27	1.13	0.60	0.28	0.09



Question 13 - Response A

(1)

(a) $H = 60 - 2(t-20)^2$

$H = 60 - 2(t^2 - 40t + 400)$

$H = 60 - (2t^2 - 80t + 800)$

$H = 60 - 2t^2 + 80t - 800$

$H = -2t^2 + 80t - 740$

(b) $H = 60 - 2(40-20)^2$

$H = 60 - 2(20)^2$

$H = -740$ ← 740 m

(c) 29 m.

(d) suggests that the carriage doesn't reach great height super quickly as it only moves for a short time anyways

1/7 marks

Part (a)

B1: $a = 60$ seen (within the equation of the model)

M0: No attempts to substitute $t = 0$, $H = 2$ into the equation.

A0: Follows M0

Part (b)

B0: 740 is incorrect.

Part (c)

M0: Neither 180 or 31 (or π) are seen.

A0: Follows M0

Part (d)

B0: This response does not satisfy the requirements of any of the bullet points in the mark scheme notes.

Question 13 - Response B

a) $t=0$ $H=2$ $\quad 2 = a - b(0-20)^2 \quad 2 = a - b(400)$

$(20, a)$ $\quad t=20$ $H=a$ $\quad a - b(20-20)^2 = 60 \quad a = 60$

$2 = 60 - 400b \quad b = 0.145$

$H = 60 - 0.145(t-20)^2$

b) $60 - 0.145(40-20)^2 = 2$

c) $H = 29(\cos 9t \cos\alpha - \sin 9t \sin\alpha) + B \quad t=0 \ H=2$

$2 = 29(\cos 0 \cos\alpha - \sin 0 \sin\alpha) + B \quad 2 = 29\cos\alpha + B$

d) the first was is linear while the second one is circular which is a better ~~[illegible]~~ model for a carriage that doo multiple circuits

5/7 marks

Part (a)

B1: $a = 60$ seen

M1: Attempts to find b by substituting in $t = 0$, $H = 2$ and their a, proceeding to a value for b

A1: $H = 60 - 0.145(t-20)^2$

Part (b)

B1: 2 condone lack of units (cao)

Part (c)

M0: Neither 180 or 31 (or π) are seen.

A0: Follows M0

Part (d)

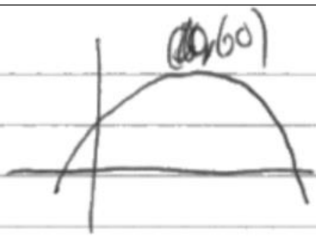
B1: They identify that the alternative model is circular (condoned as meaning cyclical) which is better for multiple circuits which satisfies the first bullet point in the notes. The reference to the linear model is ignored as this did not contradict the alternative model comments and felt that this candidate had demonstrated sufficiently an advantage of the alternative model.

Question 13 - Response C

a) $H = a - b(t-20)^2$

when $t=20$, $H=60$

$\therefore a = 60$



$H = 60 - b(t-20)^2$

at $t=0$, $H=2$

$2 = 60 - b(-20)^2$ $\quad -400b = -58$

$400b = 58 \quad b = \frac{29}{200}$

$H = 60 - \frac{29}{200}(t-20)^2$

b) when $t = 40$

$H = 60 - \frac{29}{200}(40-20)^2$

$H = 60 - \frac{29}{200}(20)^2 = 2\text{m}$

c) $H = 29\cos(9t + \alpha) + \beta$

Max of $29\cos(9t + \alpha) = 29$

Max of rollercoaster $= 60$

$29 + \beta = 60 \quad \beta = 31$

$H = 29\cos(9t + \alpha) + 31$

at $t = 0$, $H = 2$

$2 = 29\cos\alpha + 31$

$-29 = 29\cos\alpha$

$\cos\alpha = -1 \qquad \alpha = 180^\circ$

$H = 29\cos(9t + 180) + 31$

d) at $t = 120$ the height is predicted to be ~~40m~~ −1590 which is not ~~possible given a horizontally flat ground~~. likely and very extreme. The other model predicts a height of −16m which is closer to the ground making the alternative more suitable.

6/7 marks

Part (a)

B1: $a = 60$ seen.

M1: Attempts to find b by substituting in $t = 0$, $H = 2$ and their a, proceeding to a value for b

A1: $H = 60 - \frac{29}{200}(t-20)^2$

Part (b)

B1: 2 (cao)

Part (c)

M1: First scored for $(\beta =)\ 31$ (line 4 of part (c))

A1: Fully correct equation. (The lack of degrees symbol is condoned).

Part (d)

B0: They state that the alternative model predicts a height of −16m which is incorrect.

Examiner comments

Calculations were not expected in part (d). If values were used as part of an argument, then they had to be correct. Therefore, if a candidate used an incorrect model to find a value in part (d) then this mark could not have been scored.

Question 14

Introduction | Question | Mark Scheme | Examiner Comments | Performance | Response A | Response B | Response C

Question 14 - Introduction

This was a short question requiring candidate to use algebra to prove that $(n+1)^3 - n^3$ is odd for all $n \in \mathbb{N}$, typically by setting up cases for odd and even integers.

Question 14 - Question

14. Prove, using algebra, that

$$(n + 1)^3 - n^3$$

is odd for all $n \in \mathbb{N}$

(4)

(Total for Question 14 is 4 marks)

Question 14 - Mark Scheme

Question	Scheme	Marks	AOs
14	When n is even: $(2k+1)^3-(2k)^3=8k^3+12k^2+6k+1-8k^3=6k(2k+1)+1$ $\Rightarrow$ which is odd	M1	3.1a
	or When n is odd: $(2k+2)^3-(2k+1)^3=8(k^3+3k^2+3k+1)-(8k^3+12k^2+6k+1)=6k(2k+3)+7$ $\Rightarrow$ which is odd	A1	2.2a
	When n is even: $(2k+1)^3-(2k)^3=8k^3+12k^2+6k+1-8k^3=6k(2k+1)+1$ $\Rightarrow$ which is odd **and** When n is odd: $(2k+2)^3-(2k+1)^3=8(k^3+3k^2+3k+1)-(8k^3+12k^2+6k+1)=6k(2k+3)+7$ $\Rightarrow$ which is odd	dM1	2.1
	Hence odd for all $n\left(\in\mathbb{N}\right)$ *	A1*	2.4
			(4 marks)

Q14

i

?

A

B

C

Question 14 - Examiner Comments

Candidates were required to prove the statement: $(n+1)^3 - n^3$ is odd for all natural numbers, n. This was similar to some of the proof questions that have been on previous papers, so it was pleasing to see the majority of candidates were able to make more progress on this type of question this time.

A large number of candidates tried to start correctly by substituting $n = 2k$ and $n = 2k+1$(or $n = 2k-1$) for even and odd numbers; this was the most likely method to achieve full marks. Candidates would often produced the correct expressions which were written in the form 2(......)+1 and then conclude appropriately. The final mark was often lost in many of the complete attempts as at least one element of the proof was either missing or had errors. Some candidates only proved one case (odd or even only) and some candidates did not include an overall conclusion, which was necessary for the final mark.

Most common errors seen in expansions were from $(2k)^3 = 2k^3$ and this often led to candidates achieving a cubic expression rather than a quadratic. Some used $2n$ and $2n+1$ which was penalised by losing the final mark. Errors in expanding and simplifying the algebra were common: expanding triple brackets was the most common approach, although the binomial expansion was also seen, on occasion.

The other common approach was from an attempt of algebra with logic. It was common to see candidates expand and simplify the expression to achieve a three-term quadratic but only a few went on to factorise their quadratic as required, and even fewer explained the logic for why $n(n+1)$ was even correctly.

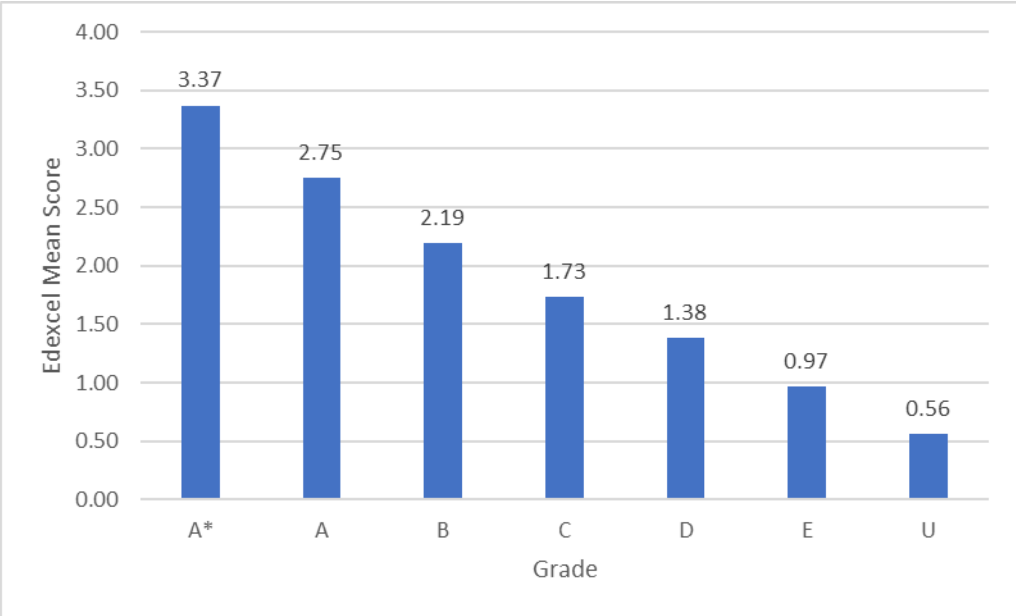
A significant number of candidates tried to expand and simplify the given expression and usually achieved $3n^2 + 3n + 1$ scoring the first 2 marks, but very few got beyond this without making any logical arguments at all or did not factorise the expression. e.g. $3n(n+1)+1$ which gained 3 marks. Success was also seen when the given expression was expanded and simplified to $3n^2 + 3n + 1$ and then candidates substituted $n = 2k$ and $n = 2k+1$ to much the same result as the main method seen in the mark scheme.

A number of candidates tried proof by contradiction, but these almost all fell into being marked either from the main scheme or by an expanding and factorising attempt. Whilst the question did require candidates to use algebra, credit was given for those candidates who did just try to apply logic with a maximum score of 2 marks. It was pleasing, however, to see that most candidates had appreciated the demand of the question and attempted some manipulation. A few candidates tried the Further Maths Method "proof by induction", but most could not complete the proof correctly.

Q14
A
B
C

Question 14 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
2.19	4	55%	3.37	2.75	2.19	1.73	1.38	0.97	0.56



Question 14 - Response A

$n = 2x$ (when n = even)

$(2x+1)^3 - (2x)^3$

$(2x+1)(2x+1)$

$(4x^2+4x+1)(2x+1) = 8x^3+4x^2+8x^2+4x+2x+1$

$8x^3+12x^2+6x+1$

$2(4x^3+6x^2+3x+\frac{1}{2})$

Not divisible by 2 so is ODD

$n = 2x+1$ (n = odd)

$(2x+1+1)^3 - (2x+1)^3$ $\quad$ $(2x+1)^3$

$2x+1+1$ $\quad$ $= 8x^3+12x^2+6x+1$

$(2x+2)^3$

$(2x+2)(2x+2)$

$(4x^2+8x+4)(2x+2)$

$8x^3+8x^2+16x^2+16x+\cancel{4x}\ 8x+8$

$8x^3+24x^2+\cancel{16}\ 24x+8$

Not divisible by 2 so is ODD

$= 8x^3+24x^2+24x+8 + 8x^3+12x^2+6x+1$

$= 16x^3+36x^2+36x+9$

$2(8x^3+18x^2+18x+\frac{9}{2})$

0/4 marks

“Algebraic method using $n = 2x$ and $n = 2x+1$”

M0: Attempts $(n+1)^3 - n^3$ when $n = 2x$ (and also when $n = 2x+1$) but only multiplies out the first bracket in each case. As a result they never simplify to achieve a three-term quadratic.

A0dM0A0*: Follows M0

Question 14 - Response B

$(n+1)^3 - n^3$

Assumption:

~~Th~~ There exists a real value of n such that $(n+1)^3 - n^3$ is even.

$(n+1)^3 - n^3 = 2k$

$n+1-n = \sqrt[3]{2k}$

$\sqrt[3]{2k} = 1$

$k = \frac{1}{2}$

$(n+1)^3 = (n+1)(n+1)^2$

$= (n+1)(n^2+2n+1)$

$= n^3+2n^2+n+n^2+2n+1$

$= n^3+3n^2+3n+1$

$n^3+3n^2+3n+1 - n^3 = 2k$

$3n^2+3n+1 = 2k$

cannot be factorised, solving for n doesn't give a real number. ∴ n cannot give a a even number.

This is also ~~because~~ due to the $+1$ after multiplying by 3 and squaring the first term

2/4 marks

Q14 A B C

The candidate appears to have attempted a proof by contradiction.

M1: Attempts to multiply out the brackets and simplifies to achieve a three-term quadratic. (line 11)

A1: $3n^2+3n+1$ (line 11)

dM0: Sets the quadratic equal to $2k$ but makes no further progress with attempting to factorise.

A0*: Follows dM0

Examiner comments

This solution would also score two marks if you applied the algebraic with logic approach.

Question 14 - Response C

Let my odd numbers be $2n+1$

$\cancel{(2n+1)^3 - 2n}$

$(2n+1+1)^3 - (2n+1)^3$

$= (2n+2)^3 - (2n+1)^3$

$= (2n+2)(2n+2)(2n+2) - (2n+1)(2n+1)(2n+1)$

$= \cancel{8n} (4n^2+8n+4)(2n+2) - (4n^2+4n+1)(2n+1)$

$= 8n^3 + 8n^2 + 16n^2 + 16n + 8n + 8 - (4n^2+4n+1)(2n+1$

$8n^3 + 24n^2 + 24n + 8 - (4n^2+4n+1)(2n+1)$

$8n^3 + 24n^2 + 24n + 8 - (8n^3 + 4n^2 + 8n^2 + 4n + 2n + 1)$

$\cancel{8n^3} + 24n^2 + 24n + 8 - \cancel{8n^3} - 12n^2 - 6n - 1$

$12n^2 - 18n + 7$

$12n^2 - 18n + 6 + 1$

$2(6n^2 - 9n + 3) + 1$

even but added by a 1 $\therefore$ its odd

let ~~br~~ my even number be $2m$

$(2m+1)^3 - (2m)^3$

$= 8m^3 + 12m^2 + 6m + 1 - 8m^3$

$= 12m^2 + 6m + 1$

$= 2(6m^2 + 3m) + 1$

even when $\times$ by 2 but added by a 1 $\therefore$ odd.

~~So~~ so $(n+1)^3 - n^3$ is odd for all $n \in \mathbb{N}$

3/4 marks

"Algebraic method using $n=2m$ and $n=2n+1$"

Here the candidate makes an error in their attempt at $n=2n+1$ (we condone use of $n=2n+1$ for all but the final A mark). Their attempt at $n=2m$ scores more marks so we mark this part of the solution first.

M1: Finds $(n+1)^3-n^3$ when $n=2m$ and attempts to multiply out and simplify to achieve a three-term quadratic.

A1: Complete argument for $n=2m$ showing the result is odd including a correct simplified quadratic expression, reason, and conclusion.

dM1: Finds $(n+1)^3-n^3$ when $n=2n+1$ (condoned) and attempts to multiply out and simplify to achieve a three-term quadratic.

A0: Incorrect simplified quadratic. Note, this mark cannot be scored as they set $n=2n+1$

Question 15

Introduction | Question | Mark Scheme | Examiner Comments | Performance | Response A | Response B | Response C

Question 15 - Introduction

This final question on the paper required candidates to typically differentiate using the product rule, quotient rule and chain rule in part (a). In part (b), candidates were expected to use their result from part (a) to set equal to zero and rearrange to achieve the given result. Candidates, irrespective of their success in these first two parts, were able to draw a staircase diagram in part (c), use the iteration formula in part (d) and use a suitable interval and a suitable function in part (e) to show that $\alpha = -0.432$ correct to 3 decimal places.

Question 15 - Question

15. A curve has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}} \qquad x > \ln\sqrt[3]{2}$$

(*a*) Show that

$$\mathrm{f}'(x) = \frac{7e^x(e^{3x}(2-x) + Ax + B}{2(e^{3x}-2)^{\frac{3}{2}}}$$

where A and B are constants to be found.

(5)

(*b*) Hence show that the x coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

(2)

The equation $x = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ has two positive roots α and β where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for α and β

Diagram 1 shows a plot of part of the curve with equation $y = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ and part of the line with equation $y = x$

Using Diagram 1 below

(c) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for β

(1)

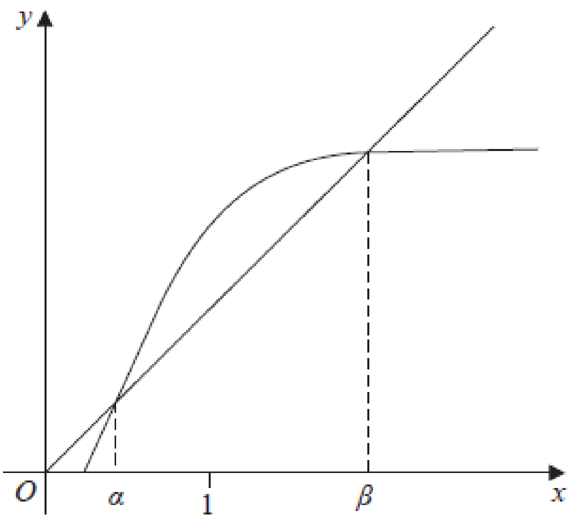


Diagram 1

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

(d) (i) the value of x_2

(ii) the value of β

(3)

Using a suitable interval and a suitable function that should be stated

(e) show that $\alpha = 0.432$ to 3 decimal places.

(2)

(Total for Question 15 is 13 marks)

Question 15 - Mark Scheme

Question	Scheme	Marks	AOs
15(a)	$...xe^{x} + ...e^{x}$	M1	1.1b
	$k\left(xe^{x} + e^{x}\right)$	A1	1.1b
	$\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right) = \frac{1}{2} \times 3e^{3x}\left(e^{3x}-2\right)^{-\frac{1}{2}}$	B1	1.1b
	$(f'(x) =)\ \frac{(e^{3x}-2)^{\frac{1}{2}}("7"xe^{x} + "7"e^{x}) - "\frac{3}{2}"e^{3x}(e^{3x}-2)^{-\frac{1}{2}} \times "7"xe^{x}}{e^{3x}-2}$	dM1	2.1
	$f'(x) = \frac{7e^{x}\left(e^{3x}(2-x) - 4x - 4\right)}{2(e^{3x}-2)^{\frac{3}{2}}}$	A1	1.1b
		(5)	
(b)	$e^{3x}(2-x) - 4x - 4 = 0 \Rightarrow x(...e^{3x} \pm ...) = ...e^{3x} \pm ...$	M1	1.1b
	$\Rightarrow x = \frac{2e^{3x}-4}{e^{3x}+4}$ *	A1*	2.1
		(2)	
(c)	Draws a vertical line $x = 1$ up to the curve then across to the line $y = x$ then up to the curve finishing at the root (need to see a minimum of 2 vertical and horizontal lines tending to the root)	B1	2.1
		(1)	
(d)(i)	$x_2 = \frac{2e^{3}-4}{e^{3}+4} = 1.5017756...$	M1	1.1b
	$x_2 =$ awrt 1.502	A1	1.1b
(ii)	$\beta = 1.968$	dB1	2.2b
		(3)	
(e)	$h(x) = \frac{2e^{3x}-4}{e^{3x}+4} - x$ $h(0.4315) = -0.000297...$ $h(0.4325) = 0.000947...$	M1	3.1a
	Both calculations correct and e.g. states: • There is a change of sign • e.g $f'(x)$ is continuous • $\alpha = 0.432$ (to 3dp)	A1cao	2.4
		(2)	
		(13 marks)	

Question 15 - Examiner Comments

This question brought together a variety of topics and was appropriately placed at the end of the paper. This was very good at discriminating between candidates at the higher end, but it was also pleasing to see a good number of weaker candidates who were still able to attempt later parts such as part (c) and part (d) to a pleasing degree of success.

In part (a) a significant proportion of candidates understood that they needed to use the product rule on $7xe^x$ and most candidates who did, applied it correctly. A common error was just to write $7e^x$ or $7xe^x$. Most candidates were unable to achieve the B1 mark, with most not applying the chain rule and so missing terms or making errors with powers. Candidates usually used their expressions correctly with the quotient rule, but few candidates scored the final method mark due to not having the correct expressions. Most candidates chose to use the quotient rule method, but those who used product rule instead, and initially rewrote the denominator as $\left(e^{3x}-2\right)^{-\frac{1}{2}}$, were generally successful in achieving the first four marks. Some, however, omitted brackets in their expression which lost the final two marks. The final A1 mark was achieved by very few candidates as most did not complete the rearrangement process. Some candidates adjusted their answer by using the given answer in part (b) to work out what the values of A and B should be. Whilst candidates should not typically use later information in earlier parts, this was condoned, as in some cases it would have been too difficult to determine whether this had been done so the benefit of the doubt was given. To achieve the correct form in part (a) from correct differentiation deserved full credit, regardless of the route to the final form required. It should be noted, however, that there were a few who integrated instead of differentiating.

In part (b) very few candidates attempted a solution having not completed a solution to part (a). Some determined candidates attempted a solution using A and B and so were able to achieve the method mark. Some candidates attempted a solution by using the given solution to work out what the values of A and B should be and then attempted a solution with those values. A minority of candidates who attempted a solution did not show all the steps required for a show that question.

In part (c), the majority of candidates scored the mark for a correct diagram, although some started their initial line higher up than the x-axis, which was condoned on this occasion. Several candidates used a cobweb diagram, and for others there were often lines drawn to the left of $x=1$. Only a handful of candidates put arrows on their lines, but their omission was, again, condoned.

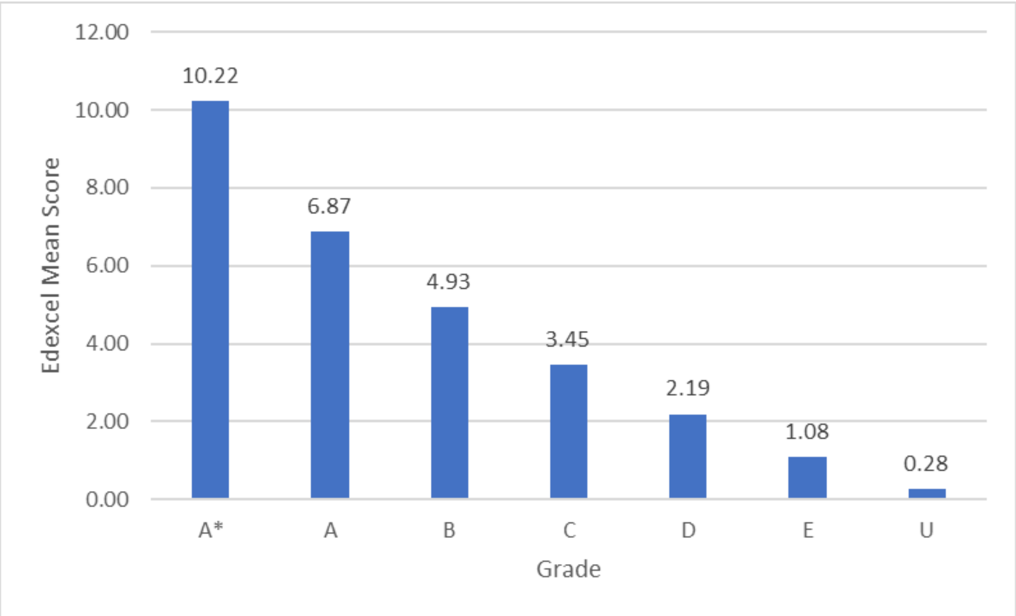
Part (d), for many candidates who had not scored in or attempted earlier parts, was an opportunity to score full marks relatively easily. This was very well done. Occasionally, rounding x_2 to 1.50 rather than 1.502 was seen. However, those who wrote down an unrounded answer which rounded to 1.502 first were able to get the A mark. Candidates who showed the value embedded in the formula and then did not achieve 1.502 were able to get the M mark, too. A minority of candidates skipped 1.502 to go to the next term, 1.873 but this still was able to score the method mark as this was still evidence that the iterative formula had been used. The correct answer $\beta = 1.968$ was widely achieved for part (b)(ii), although 1.967 was a common incorrect answer. The final mark was also lost for a small number of candidates who failed to round to 3 decimal places.

Part (e) was the final part on the paper and for many this resulted in no marks. Most of the candidates who attempted it found a suitable interval but substituted the values in the iterative formula instead of a valid equation set equal to zero. Many candidates wrote the required conclusion with no supporting evidence in an attempt to gain a mark. Some candidates, who used a valid function correctly, failed to score the final mark for not commenting that their function was continuous. A significant proportion of candidates attempted an iterative process with a different iterative formula, including attempting the Newton-Raphson method. The question required used of a suitable interval and so these methods scored no marks.

Overall, whilst this question was demanding, there were some very impressive solutions demonstrating an excellent grasp of calculus, strong algebraic skills, and the ability to provide a rigorous argument to prove given answers and results.

Question 15 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
5.21	13	40%	10.22	6.87	4.93	3.45	2.19	1.08	0.28



Question 15 - Response A

Question 15 continued

(a) $f(x) = \dfrac{7xe^x}{\cancel{ex}\,(e^{3x}-2)^{1/2}}$

$u = 7xe^x \qquad v = (e^{3x}-2)^{1/2}$

$u' = \cancel{7e}\, 7xe^x + 7e^x \qquad v' = \frac{1}{2}(e^{3x}-2) \times 3e^{3x}$

$u = 7x \qquad v = e^x$

$u' = 7 \qquad v' = e^x$

$\dfrac{\cancel{7e}\; 7xe^x + 7e^x (e^{3x}-2)^{1/2} - 7xe^x\, \frac{3}{2} e^{3x}(e^{3x}-2)}{}$

$e^x(7x+7)\cancel{a}(e^{3x}-2)^{1/2}$

$\cancel{15}$ $(7xe^x)($

Question 15 continued

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

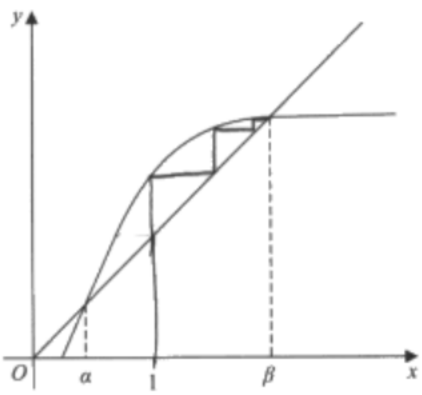
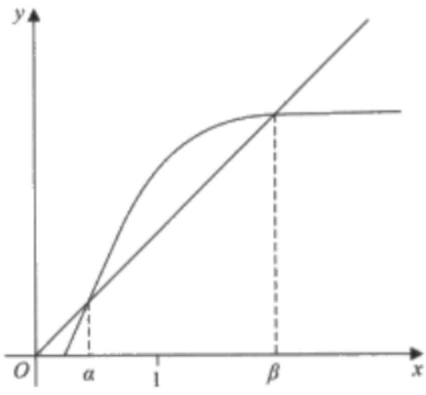


Diagram 1



copy of Diagram 1

15) $\dfrac{7xe^x}{\sqrt{e^{3x}-2}}$

$U = 7xe^x \qquad V = (e^{3x}-2)^{1/2}$

$u' = e^x(7x+7) \qquad v = 1/$

Q15 i ? A B C

(a) $\frac{2e^{3x_n}-4}{e^{3x_n}+4}$ $x_1 = 1$

(i) $x_2 = \frac{2e^{3(1)}-4}{e^{3(1)}+4} = 1.501$

(ii)

~~3~~ β = ~~1.980~~ 1.968

$x_2 = \frac{2e^{3(1.98)}-4}{e^{3(1.98)}+4} = 1.873$

$x_4 = \frac{2e^{3(1.873)}-4}{e^{3(1.98)}+4} = 1.957$

Question 15 continued

~~3.5~~

e) f(0.4315) = ~~-5...3~~ 0.431

f(0.4325) = 0.433

5/13 marks

Part (a)

M1: Attempts the product rule on $7xe^x$ achieving an expression of the form $...xe^x \pm ...e^x$. (line 3). The first two marks could be scored whether the 7 was present or not.

A1: $k\left(xe^x + e^x\right)$ (line 3)

B0: Incorrect, the power $-\frac{1}{2}$ is missing.

dM0: Incomplete attempt at the quotient rule.

A0: Follows dM0

Part (b)

No attempt seen

Part (c)

B1: Starting at $x_1 = 1$ draws at least 2 sets of vertical and horizontal lines tending to β

Part (d)(i)

M1: Substitutes 1 into the iterative formula (embedded values are sufficient). Also implied by awrt 1.50

A0: Not awrt 1.502

Q15

i ? A B C

Part (d)(ii)

dB1: 1.968 cao (this could only be scored as M1 had been awarded in (d)(i))

Part (e)

M0: Does not attempt to substitute $x = 0.4315$ and 0.4325 into a suitable function.

A0: Follows M0

Question 15 - Response B

Question 15 continued

a) $f'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$ $\qquad \dfrac{f(x)}{g(x)} = f(x)$

$7xe^x \quad f'(x) = \quad u = 7x \quad v = e^x$

$\dfrac{du}{dx} = 7 \quad \dfrac{dv}{dx} = e^x \qquad \dfrac{x}{y^{\frac{3}{2}}} \quad \dfrac{x}{\sqrt{y^3}}$

$= 7xe^x + 7e^x$

$\sqrt{e^{3x}-2} = (e^{3x}-2)^{\frac{1}{2}} \quad g'(x) = \frac{1}{2}(e^{3x}-2)^{-\frac{1}{2}} \times 3e^{3x}$

$= \frac{3}{2}e^{3x}(e^{3x}-2)^{-\frac{1}{2}}$

$-\frac{1}{\sqrt{2}}$

$(g(x))^2 = e^{3x}-2 \qquad \frac{3}{2}e^{3x} \times e^{-\frac{1}{2}x} \quad -2^{-\frac{1}{2}}$

$$\frac{(7xe^x + 7e^x)(\sqrt{e^{3x}-2}) - (7xe^x)(\frac{3}{2}e^{3x}(e^{3x}-2)^{-\frac{1}{2}})}{e^{3x}-2}$$

$$\frac{(7xe^x + 7e^x)(e^{3x}-2)^{\frac{1}{2}} - (7xe^x)(\frac{3}{2}e^{\frac{3}{2}x} - \frac{3\sqrt{2}}{4}e^{3x})}{e^{3x}-2}$$

$$\frac{7e^x(e^{3x}(2-x)+}{2(e^{3x}-2)^{\frac{3}{2}}}$$

Q15

A

B

C

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

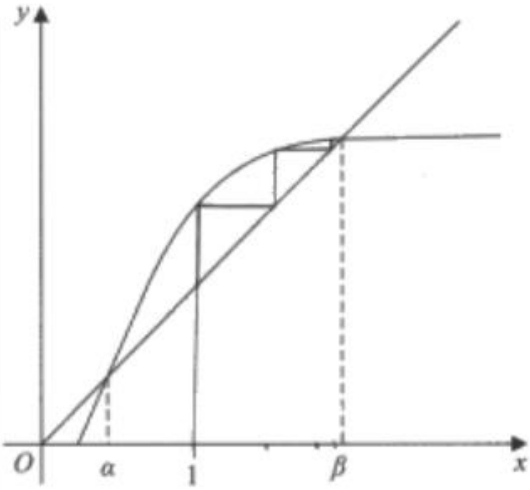
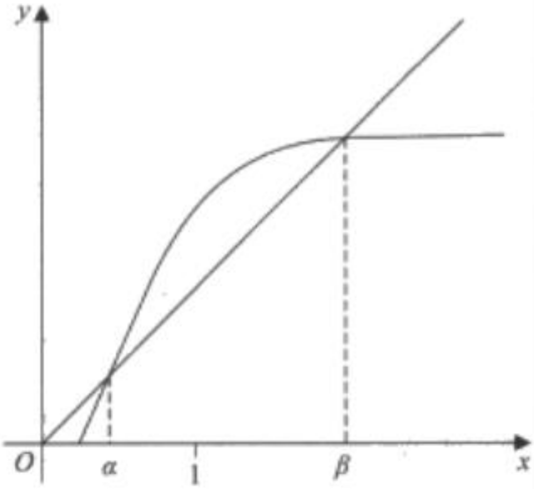


Diagram 1



copy of Diagram 1

$x_2 = 1.502 \quad x_3 =$

b) $\dfrac{7e^{x}\left(e^{3x}(2-x)\right)}{2\left(e^{3x}-2\right)^{3/2}} = 0$

d) i) $x = \dfrac{2e^{3}-4}{e^{3}+4} = 1.502$ (3dp)

ii) $\beta = 1.968$ (3dp)

e) $x_1 = 0.1$ $\qquad \alpha = -0.958$

3 solutions $\rightarrow 1.968$

$\rightarrow 0.432$

$\rightarrow -0.958$

8/13 marks

Q15

A

B

C

Part (a)

M1: Attempts the product rule on xe^x achieving an expression of the form $...xe^x \pm ...e^x$. (line 4). Again, it did not matter for the first two marks if the 7 was included or not.

A1: $k\left(xe^x + e^x\right)$ (line 4)

B1: $\frac{1}{2} \times 3e^{3x}\left(e^{3x} - 2\right)^{-\frac{1}{2}}$ or equivalent (line 5)

dM1: Attempts the quotient rule and achieves the required form.

A0: Incorrect

Part (b)

M0: Sets their answer to part (a) = 0 but makes no further progress.

A0*: Follows M0

Part (c)

B1: Starting at $x_1 = 1$ draws at least 2 sets of vertical and horizontal lines tending to β

Part

M1: Substitutes 1 into the iterative formula (embedded values are sufficient but also implied by 1.502)

A1: awrt 1.502

Part (d)(ii)

dB1: 1.968 cao

Part (e)

M0: Does not attempt to substitute $x = 0.4315$ and 0.4325 into a suitable function.

A0: Follows M0

Question 15 - Response C

a) $f(x) = \dfrac{7xe^x}{\sqrt{e^{3x}-2}}$

$u = 7xe^x \qquad v = \sqrt{e^{3x}-2}$

$u' = 7e^x + 7xe^x \qquad v' = \frac{3}{2}e^{3x}(e^{3x}-2)^{-1/2}$

$\therefore f'(x) = \dfrac{(7e^x+7xe^x)(\sqrt{e^{3x}-2}) - 7xe^x(\frac{3}{2}e^{3x}(e^{3x}-2)^{-1/2})}{(e^{3x}-2)}$

$= \dfrac{7e^x(1+x)(e^{3x}-2)^{1/2} - 7e^x(x(\frac{3}{2}e^{3x}(e^{3x}-2)^{-1/2}))}{(e^{3x}-2)}$

$= \dfrac{7e^x(1+x)(e^{3x}-2) - 7e^x(x)(\frac{3}{2}e^{3x})}{(e^{3x}-2)^{3/2}}$

$= \dfrac{7e^x[(1+x)(e^{3x}-2) - x(\frac{3}{2}e^{3x})]}{(e^{3x}-2)^{3/2}}$

$= \dfrac{7e^x[(2+2x)(e^{3x}-2) - x(3e^{3x})]}{2(e^{3x}-2)^{3/2}}$

$= \dfrac{7e^x[2e^{3x}-4+2xe^{3x}-4x-3xe^{3x}]}{2(e^{3x}-2)^{3/2}}$

$= \dfrac{7e^x[e^{3x}(2-x)-4x-4]}{2(e^{3x}-2)^{3/2}}$

$A=-4 \quad B=-4$

b) $\dfrac{7e^x(e^{3x}(2-x)-4x-4)}{2(e^{3x}-2)^{3/2}} = 0$ ($f'(x)=0$ at stationary point)

$\Rightarrow e^{3x}(2-x)-4x-4=0$ (as $e^x \neq 0$)

$\Rightarrow 2e^{3x}-xe^{3x}-4x-4=0$

$\Rightarrow 2e^{3x}-4 - x(e^{3x}+4)=0$

$\Rightarrow 2e^{3x}-4 = x(e^{3x}+4)$

$\Rightarrow x = \dfrac{2e^{3x}-4}{e^{3x}+4}$ shown

Q15

i ? A B C

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

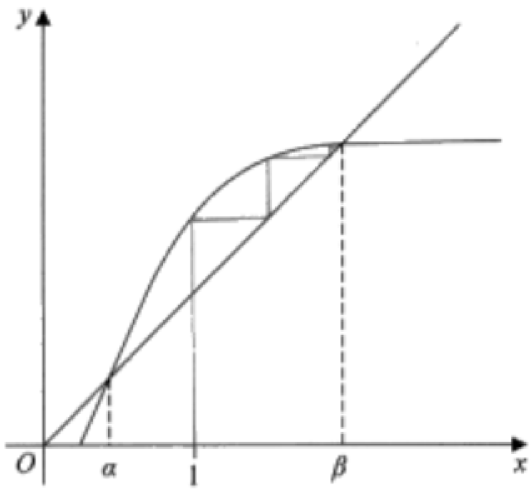
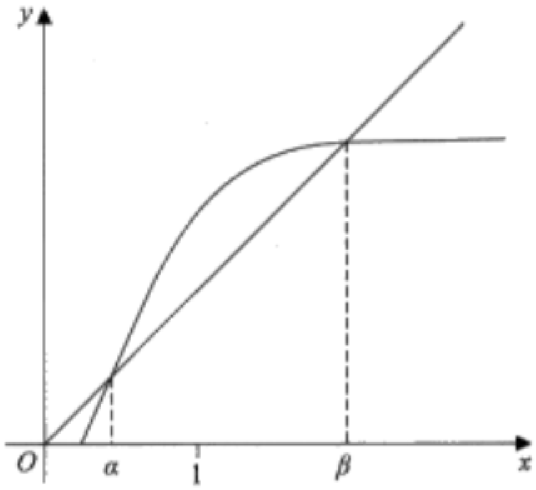


Diagram 1



copy of Diagram 1

d) i) $x_1 = 1$

$x_2 = \dfrac{2e^{3(1)} - 4}{e^{(3)(1)} + 4}$

$= 1.501775\ldots$

$= 1.502$ (3 dp)

ii) $x_3 = 1.873$

$x_4 = 1.957$

$x_5 = 1.967$

$x_6 = 1.967$

$x_7 = 1.968$

$x_8 = 1.968$

$x_9 = 1.968$ (all to 3 dp)

$\therefore \beta = 1.968$ (3dp)

e) $\cancel{g(x) = \dfrac{2e^{3x} - 4}{e^{3x}}}$

$g(x) = \dfrac{7e^{x}\left(e^{3x}(2-x) - 4x - 4\right)}{2(e^{3x}-2)^{3/2}}$

in interval $[0.4315, 0.4325)$

$g(0.4315) = \dfrac{7e^{0.4315}\left(e^{3(0.4315)}(2-0.4315)-4(0.4315)-4\right)}{2\left(e^{3(0.4315)}-2\right)^{3/2}}$

$= -5.789 \times 10^{-3} < 0$

(4sf)

$g(0.4325) = 0.01831 > 0$

(4sf)

$\therefore$ as the function $g(x)$ is continuous in the interval $[0.4315, 0.4325]$, and there is a change of sign $g(x)$ must have a root in this interval, which is $\alpha = 0.432$ to 3 decimal places.

$g(x) = f'(x)$ at $x = \alpha$,

$\Rightarrow f(x)$ must have a stationary point, $\alpha = 0.432$.

13/13 marks

Part (a)

M1: Attempts the product rule on $7xe^x$ achieving an expression of the form $...xe^x \pm ...e^x$.

A1: $k\left(xe^x + e^x\right)$ (line 4)

B1: $\dfrac{1}{2} \times 3e^{3x}\left(e^{3x} - 2\right)^{-\frac{1}{2}}$ or equivalent (line 5)

dM1: Attempts the quotient rule and achieves the required form.

A0: Incorrect

Part (b)

M1: Sets $e^{3x}(2-x) - 4x - 4$ equal to zero, collects terms in x on one side of the equation and non x terms on the other and attempts to factorise the side with x as a common factor.

A1*: Achieves the given answer with no errors including invisible brackets.

Part (c)

B1: Starting at $x_1 = 1$ draws at least 2 sets of vertical and horizontal lines tending to β

Part (d)(i)

M1: Substitutes 1 into the iterative formula (embedded values is sufficient)

A1: awrt 1.502

Part (d)(ii)

dB1: 1.968 cao

Part (e)

M1: Attempts to substitute $x = 0.4315$ and 0.4325 into a suitable function and gets one value correct.

A1: Correct calculations, a statement that there is a change in sign and that their function is continuous, and a minimal conclusion